VITEEE Practice Problems

Exam Date 10^{th} to 21^{st} April

Time : 2 Hours 30 Minutes | *Max. Marks : 120*

PHYSICS

1. Two spherical conductors B and C having equal radii and carrying equal charges in them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius but uncharged is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is

(a) $\frac{F}{4}$ (b) $\frac{3F}{4}$

(c) $\frac{F}{8}$ (d) $\frac{3F}{8}$

2. Two point charges A and B of values $+5 \times 10^{-9}$C and $+3 \times 10^{-9}$ C are kept 6 cm apart in air. When the charge B is moved by 1 cm towards charge A, then work done is equal to

(a) 4.5×10^{-6} J (b) 4.5×10^{-7} J

(c) 3.5×10^{-7} J (d) 4.5×10^{-8} J

3. The maximum wavelength of radiation that can produce photoelectric effect in certain metal is 200 nm. The maximum kinetic energy acquired by electron due to radiation of wavelength 100 nm will be

(a) 12.4 eV (b) 6.2 eV

(c) 100 eV (d) 200 eV

4. When a hydrogen atom emits a photon in going from $n = 5$ to $n = 1$, its recoil speed is almost

(a) 10^{-4} m s^{-1} (b) 8×10^{2} m s^{-1}

(c) 2×10^{-2} m s^{-1} (d) 4 m s^{-1}

5. In Newton's rings experiment, the diameter of the 15^{th} ring was found to be 0.59 cm and that of the 5^{th} ring was 0.336 cm. If the radius of the plano convex lens is 100 cm, compute the wavelength of light used.

(a) 5880 Å (b) 4880 Å

(c) 588 Å (d) 4980 Å

6. An electric dipole is placed at an angle of 30° with an electric field of intensity 2×10^{5} N C^{-1}. It experiences a torque equal to 4 N m. Calculate the charge on the dipole if the dipole length is 2 cm.

(a) 2 mC (b) 4 mC

(c) 2 μC (d) 4 μC

7. A 10 μF capacitor is charged to a potential difference of 1000 V. The terminals of the charged capacitor are disconnected from the power supply and connected to the terminals of an uncharged 6 μF capacitor. What is the final potential difference across each capacitor?

(a) 167 V (b) 100 V

(c) 625 V (d) 250 V

8. The exchange particle of the electromagnetic force is the

(a) gluon (b) muon

(c) proton (d) photon

9. Radiation can be a hazard to living organisms because it

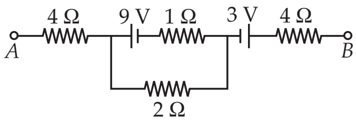
(a) produces ionization along its path of travel

(b) disrupts chemical bond

(c) generates free polyatmic ions

(d) all of the above

10. In the circuit shown in figure, potential difference between points A and B is 16 V. The current passing through 2 Ω resistance will be



(a) 2.5 A (b) 3.5 A

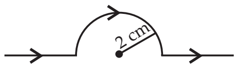
(c) 4.0 A (d) zero

11. n identical cells are joined in series with two cells A and B with reversed polarities. EMF of each cell is ε and internal resistance is r. Potential difference across cell A or B is $(n > 4)$

(a) $\frac{2\varepsilon}{n}$ (b) $2\varepsilon\left(1-\frac{1}{n}\right)$

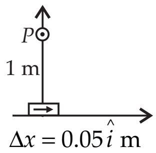
(c) $\frac{4\varepsilon}{n}$ (d) $2\varepsilon\left(1-\frac{2}{n}\right)$

12. A straight wire carrying a current of 13 A is bent into a semi-circular arc of radius 2 cm as shown in figure. The magnetic field is 1.5×10^{-4} T at the centre of arc, then the magnetic field due to straight segment is



(a) 1.5×10^{-4} T (b) 2.5×10^{-4} T
(c) zero (d) 3.0×10^{-4} T

13. An element of $0.05\,\hat{i}$ m is placed at the origin as shown in figure which carries a large current of 10 A. The magnetic field at a distance of 1 m in perpendicular direction is



(a) 4.5×10^{-8} T (b) 5.5×10^{-8} T
(c) 5.0×10^{-8} T (d) 7.5×10^{-8} T

14. Two Nicols are oriented with their principal planes making an angle of 60°. Then the percentage of incident unpolarised light which passes through the system is
(a) 100 (b) 50
(c) 12.5 (d) 37.5

15. A helium nucleus makes a full rotation in a circle of radius 0.8 m in 2 s. The value of the magnetic field induction B in tesla at the centre of circle will be

(a) $2 \times 10^{-19}\,\mu_0$ (b) $\frac{10^{-19}}{\mu_0}$
(c) $10^{-19}\,\mu_0$ (d) $\frac{2 \times 10^{-19}}{\mu_0}$

16. A torque required to hold a small circular coil of 10 turns, 2×10^{-4} m^2 area and carrying 0.5 A current in the middle of a long solenoid of 10^3 turns/m carrying 3 A current, with its axis perpendicular to the axis of the solenoid, is
(a) $12\pi \times 10^{-7}$ N m (b) $6\pi \times 10^{-7}$ N m
(c) $4\pi \times 10^{-7}$ N m (d) $2\pi \times 10^{-7}$ N m

17. If a charged particle of charge 5 μC and mass 5 g is moving with constant speed 5 m s^{-1} in a uniform magnetic field B on a curve $x^2 + y^2 = 25$, where x and y are in metre. The value of magnetic field will be
(a) 1 kT along x-axis
(b) 1 kT along z-axis
(c) 5 kT along the x-axis
(d) 1 kT along any line in the x-y plane

18. Light of certain colour has 2000 waves to the millimetre in air. What will be the wavelength of this light in a medium of refractive index 1.25?
(a) 1000 Å (b) 2000 Å
(c) 3000 Å (d) 4000 Å

19. A conducting circular loop of radius a and resistance R is kept on a horizontal plane. A vertical time varying magnetic field $B = 2t$ is switched on at time $t = 0$. Then
(a) power generated in the coil at any time t is constant
(b) flow of charge per unit time from any section of the coil is constant
(c) total charge passed through any section between time $t = 0$ to $t = 2$ s is $\left(\frac{4\pi a^2}{R}\right)$
(d) all of the above.

20. A coil in the shape of an equilateral triangle of side l is suspended between the pole pieces of a permanent magnet, such that $\vec{B}$ is in plane of the coil. If due to a current I in the triangle, a torque $\vec{\tau}$ act on it, the side l of the triangle is

(a) $\frac{2}{\sqrt{3}}\left(\frac{\tau}{BI}\right)$ (b) $2\left(\frac{\tau}{\sqrt{3}BI}\right)^{1/2}$
(c) $\frac{2}{\sqrt{3}}\left(\frac{\tau}{BI}\right)^{1/2}$ (d) $\frac{1}{\sqrt{3}}\left(\frac{\tau}{BI}\right)$

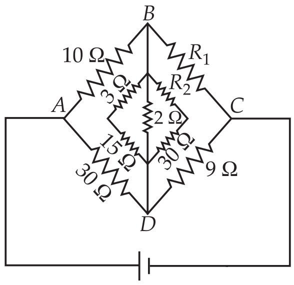
21. The magnifying power of a telescope is 9. When it is adjusted for parallel rays the distance between the objective and eyepiece is 20 cm. The focal length of lenses are
(a) 10 cm, 10 cm (b) 15 cm, 5 cm
(c) 18 cm, 2 cm (d) 11 cm, 9 cm

22. At a point on the right bisector of a magnetic dipole, the magnetic
(a) potential varies as $\frac{1}{r^2}$
(b) potential is zero at all points on the right bisector
(c) field varies as r^3
(d) field is perpendicular to the axis of dipole.

23. In an LR circuit $\upsilon = 50$ Hz, $L = 2$H, $V = 5$ V, $R = 1\ \Omega$, energy stored in inductor is
(a) 50 J (b) 25 J
(c) 3.66×10^{-4} J (d) 6.33×10^{-5} J

24. Three unequal resistors in parallel are equivalent to a resistance 1 ohm. If two of them are in the ratio 1 : 2 and if no resistance value is fractional, the largest of the three resistances in ohm is
(a) 4 (b) 6
(c) 5 (d) 12

25. In the Wheatstone bridge shown below, in order to balance the bridge, we must have



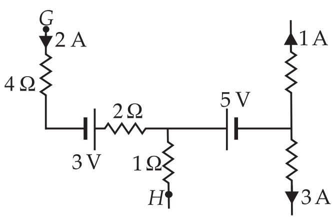
(a) $R_1 = 3\ \Omega$; $R_2 = 3\ \Omega$
(b) $R_1 = 6\ \Omega$; $R_2 = 15\ \Omega$
(c) $R_1 = 1.5\ \Omega$; R_2 = any finite value
(d) $R_1 = 3\ \Omega$; R_2 = any finite value

26. In an astronomical telescope in normal adjustment, a straight black line of length L is drawn on the objective lens. The eyepiece forms a real image of this line. The length of this image is l. The magnification of the telescope is

(a) $\frac{L}{l}$ (b) $\frac{L}{l}+1$
(c) $\frac{L}{l}-1$ (d) $\frac{L+1}{L-1}$

27. A galvanometer of resistance 50 Ω is connected to a battery of 3 V along with a resistance of 2950 Ω in series. A full scale deflection of 30 divisions is obtained in the galvanometer. In order to reduce this deflection to 20 divisions, the resistance in series should be
(a) 6050 Ω (b) 4450 Ω
(c) 5050 Ω (d) 5550 Ω

28. In the part of a circuit shown in figure, the potential difference between points G and H will be



(a) 0 V (b) 12 V
(c) 7 V (d) 3 V

29. A cell of emf 1.5 V and internal resistance 2 Ω is connected to two resistors of 5 Ω and 8 Ω in series. The potential difference across the 5 Ω resistor will be
(a) 3.3 V (b) 1 V
(c) 0.5 V (d) 0.33 V

30. In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency in the ripple would be
(a) 100 Hz (b) 70.7 Hz
(c) 50 Hz (d) 25 Hz

31. An electric charge of 8.85×10^{-13} C is placed at the centre of a sphere of radius 1 m. The electric flux through the sphere is
(a) $0.2\ N\ C^{-1}\ m^2$ (b) $0.1\ N\ C^{-1}\ m^2$
(c) $0.3\ N\ C^{-1}\ m^2$ (d) $0.01\ N\ C^{-1}\ m^2$

32. A deflection mangnetometer is adjusted in the usual way. When a magnet is introduced, the deflection observed is θ, and the period of oscillation of the needle in the magnetometer is T. When the magnet is removed, the period of oscillation is T_0. The relation between T and T_0 is
(a) $T^2 = T_0^2\cos\theta$ (b) $T = T_0\cos\theta$
(c) $T = \frac{T_0}{\cos\theta}$ (d) $T^2 = \frac{T_0^2}{\cos\theta}$

33. The most stable particle in the Baryon group is
(a) neutron (b) proton
(c) lamda particle (d) sigma particle

34. The phenomenon of polarization shows that the nature of light is
(a) particle (b) transverse
(c) longitudinal (d) dual

35. A convex lens forms an image of an object placed 20 cm away from it at a distance of 20 cm on the other side of the lens. If the object is moved 5 cm towards the lens, the image will move
(a) 5 cm towards the lens
(b) 5 cm away from the lens
(c) 10 cm towards the lens
(d) 10 cm away from the lens

36. The Fraunhofer diffraction pattern of a single slit is formed in the focal plane of a lens of focal length 1 m. The width of slit is 0.3 mm. If third minimum is formed at a distance of 5 mm from central maximum, then wavelength of light will be
(a) 5000 Å (b) 2500 Å
(c) 7500 Å (d) 8500 Å

37. Energy required to remove an electron from an aluminium surface is 4.2 eV. If light of wavelength 2000 Å falls on the surface, the velocity of faster electrons ejected from the surface is

(a) 2.5×10^{18} m s^{-1} (b) 2.5×10^{13} m s^{-1}

(c) 6.7×10^{18} m s^{-1} (d) 8.4×10^{5} m s^{-1}

38. The half-life for the α-decay of Uranium $^{238}_{92}U$ is 4.47×10^9 yr. If a rock contains sixty percent of its original $^{238}_{92}U$ atoms, its age is

[Given, log 6 = 0.778; log 2 = 0.3]

(a) 3.3×10^9 yr (b) 6.6×10^9 yr

(c) 1.2×10^8 yr (d) 5.4×10^7 yr

39. A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is

(a) 7.5° (b) 5°

(c) 15° (d) 2.5°

40. The current voltage relation of diode is given by $I = (e^{1000\ V/T} - 1)$ mA, where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?

(a) 0.05 mA (b) 0.2 mA

(c) 0.02 mA (d) 0.5 mA

CHEMISTRY

41. For a zero order reaction, the plot of concentration *vs* time is linear with

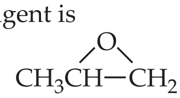
(a) positive slope and zero intercept

(b) negative slope and zero intercept

(c) positive slope and non-zero intercept

(d) negative slope and non-zero intercept.

42. The most probable structural formula for the compound whose empirical formula is C_3H_6O, and which can react with Benedict's reagent is

(a) 

(b) CH_3CH_2CHO

(c) $CH_3OCH=CH_2$

(d) $CH_2=CHCH_2OH$

43. The structural feature which distinguishes proline from other natural α-amino acids?

(a) It is optically inactive.

(b) It contains aromatic group.

(c) It contains two amino groups.

(d) It is a secondary amine.

44. Which of the following rules is not correct regarding IUPAC nomenclature of complex ions?

(a) Cation is named first and then anion.

(b) In coordination sphere, the ligands are named alphabetically.

(c) Positively charged ligands have suffix 'ate'.

(d) More than one ligand of a particular type are indicated by using di, tri, tetra, etc.

45. Which of the following leads to the formation of methyl *t*-butyl ether?

(a) $(C_2H_5)_3CONa + CH_3Cl$

(b) $CH_3ONa + (CH_3)_3CCl$

(c) $(CH_3)_3CONa + C_2H_5Cl$

(d) $(CH_3)_3CONa + CH_3Cl$

46. For a reaction, $\frac{1}{2}A \rightarrow 2B$ rate of disappearance of A is related to the rate of appearance of B by the expression

(a) $-\frac{d[A]}{dt} = 4\frac{d[B]}{dt}$ (b) $-\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$

(c) $-\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$ (d) $-\frac{d[A]}{dt} = \frac{d[B]}{dt}$

47. Which element from group 15 gives most basic compound with hydrogen?

(a) Nitrogen (b) Bismuth

(c) Arsenic (d) Phosphorus

48. Which one of the following compounds does not decolourise an acidified aqueous solution of $KMnO_4$?

(a) Sulphur dioxide

(b) Ferric chloride

(c) Hydrogen peroxide

(d) Ferrous sulphate

49. The correct order of the packing efficiency in different types of unit cells is

(a) *fcc* < *bcc* < simple cubic

(b) *fcc* > *bcc* > simple cubic

(c) *fcc* < *bcc* > simple cubic

(d) *bcc* < *fcc* > simple cubic.

50. Which of the following will exhibit highest boiling point?
(a) $CH_3CH_2OCH_2CH_2CH_3$
(b) $CH_3CH_2CH_2CH_2CH_2OH$
(c) $CH_3CH_2CH_2CH(CH_3)OH$
(d) $CH_3CH_2C(CH_3)_2OH$

51. The equivalent conductance of NaCl at concentration C and at infinite dilution are λ_C and λ_∞, respectively. The correct relationship between λ_C and λ_∞ is given as (where, the constant B is positive)
(a) $\lambda_C = \lambda_\infty + (B)\sqrt{C}$ (b) $\lambda_C = \lambda_\infty + (B)C$
(c) $\lambda_C = \lambda_\infty - (B)C$ (d) $\lambda_C = \lambda_\infty - B\sqrt{C}$

52. Which of the following reagents cannot be used to oxidise primary alcohols to aldehydes?
(a) CrO_3 in anhydrous medium
(b) $KMnO_4$ in acidic medium
(c) Pyridinium chlorochromate
(d) Heat in the presence of Cu at 573 K

53. The correct order of acidity of oxoacids of halogens is
(a) $HClO < HClO_2 < HClO_3 < HClO_4$
(b) $HClO_4 < HClO_3 < HClO_2 < HClO$
(c) $HClO < HClO_4 < HClO_3 < HClO_2$
(d) $HClO_4 < HClO_2 < HClO_3 < HClO$

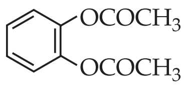
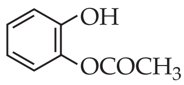
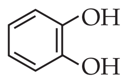
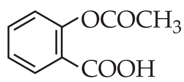
54. The data for the reaction $A + B \rightarrow C$, is

Exp.	$[A]_0$	$[B]_0$	Initial rate
1	0.012	0.035	0.10
2	0.024	0.070	0.80
3	0.024	0.035	0.10
4	0.012	0.070	0.80

The rate law corresponds to the above data is
(a) Rate = $k[A][B]^3$ (b) Rate = $k[A]^2[B]^2$
(c) Rate = $k[B]^3$ (d) Rate = $k[B]^4$

55. Reaction of cyclohexanone with dimethylamine in the presence of catalytic amount of an acid forms a compound if water during the reaction is continuously removed. The compound formed is generally known as
(a) a Schiff's base (b) an enamine
(c) an imine (d) an amine.

56. An organic compound P has 76.6% C and 6.38% H. Its vapour density is 47. It gives a characteristic colour with aq. $FeCl_3$. P when treated with CO_2 and NaOH at 140°C under pressure gives Q which on acidification gives R. R reacts with acetyl chloride to give S, which is

(a) 
(b) 
(c) 
(d) 

57. Oils and fats are esters of higher fatty acids with
(a) ethanol (b) glycol
(c) glycerol (d) methanol.

58. At 20°C, the Ag^+ ion concentration in a saturated solution of Ag_2CrO_4 is 1.5×10^{-4} mol L^{-1}. At 20°C, the solubility product of Ag_2CrO_4 would be
(a) 3.3750×10^{-12} (b) 1.6875×10^{-10}
(c) 1.6875×10^{-12} (d) 1.6875×10^{-11}

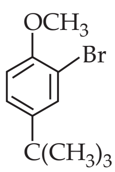
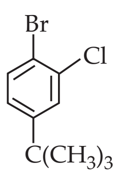
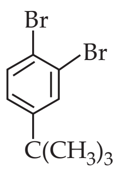
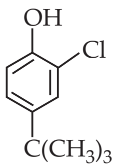
59. In corrosion of iron,
(a) an electrochemical (galvanic) cell is formed in which Fe acts as anode and the site where O_2 is reduced acts as cathode.
(b) electrons flow from anode to cathode through the metal while ions flow through the water droplets
(c) dissolved O_2 oxidises Fe^{2+} to Fe^{3+} before it is deposited as rust ($Fe_2O_3.H_2O$)
(d) all of the above takes place.

60. Phenyl isocyanide is prepared by which of the following reactions?
(a) Rosenmund's reduction
(b) Carbylamine reaction
(c) Reimer-Tiemann reaction
(d) Wurtz reaction

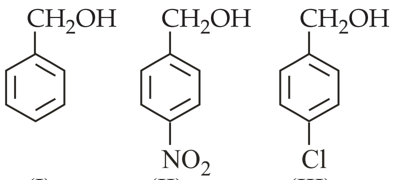
61. Zr and Hf have almost equal atomic and ionic radii because of
(a) diagonal relationship
(b) lanthanoid contraction
(c) actinoid contraction
(d) belonging to the same group.

62. Anisol $\xrightarrow[\text{anhyd. AlCl}_3]{(CH_3)_3CCl} \xrightarrow{Cl_2/FeCl_3} \xrightarrow[\text{Heat}]{HBr} X$

The product X in the above series of reactions is

(a) 
(b) 
(c) 
(d) 

63. Mark the correct increasing order of reactivity of the following compounds with HBr/HCl.



(I) (II) (III)

(a) I < II < III (b) II < I < III
(c) II < III < I (d) III < II < I

64. Which of the following is most likely structure of $CrCl_3.6H_2O$ if 1/3 of total chlorine of the compound is precipitated by adding $AgNO_3$ to its aqueous solution?
(a) $CrCl_3.6H_2O$
(b) $[Cr(H_2O)_3Cl_3](H_2O)_3$
(c) $[CrCl_2(H_2O)_4]Cl.2H_2O$
(d) $[CrCl(H_2O)_5]Cl_2.H_2O$

65. Which of the following statements is not correct?
(a) Methylamine is more basic than NH_3.
(b) Amines form hydrogen bonds.
(c) Ethylamine has higher boiling point than propane.
(d) Dimethylamine is less basic than methylamine.

66. At temperature *T*, a compound $AB_{2(g)}$ dissociates according to the reaction,

$$2AB_{2(g)} \rightleftharpoons 2AB_{(g)} + B_{2(g)}$$

with a degree of dissociation *x*, which is small compared to unity. The expression for K_p, in terms of *x* and the total pressure, *P* is
(a) $\frac{Px^3}{2}$ (b) $\frac{Px^2}{3}$
(c) $\frac{Px^3}{3}$ (d) $\frac{Px^2}{2}$

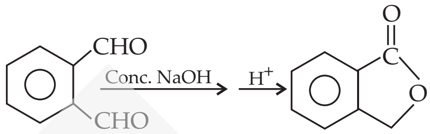
67. Which of the following is correct method to convert *p*-toluidine to *p*-toluic acid?
(a) Diazotisation, CuCN, H_2/Pd
(b) $CHCl_3$/NaOH, KCN, Sn/HCl
(c) Diazotisation, CuCN/KCN, H_2O/H^+
(d) Diazotisation, NaCN, NaOH

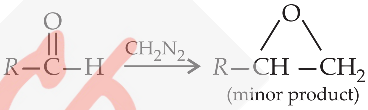
68. For the reaction given below the values of standard Gibbs free energy of formation at 298 K are given. What is the nature of the reaction?

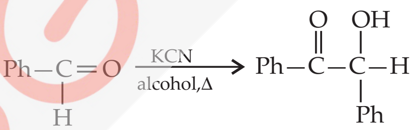
$$I_2 + H_2S \rightarrow 2HI + S$$

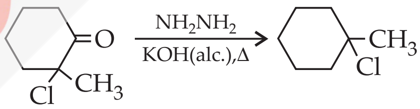
$\Delta G_f^\circ(HI) = 1.8\ kJ\ mol^{-1}$, $\Delta G_f^\circ(H_2S) = 33.8\ kJ\ mol^{-1}$
(a) Non-spontaneous in forward direction
(b) Spontaneous in forward direction
(c) Spontaneous in backward direction
(d) Non-spontaneous in both forward and backward directions

69. In which of the following reactions, the product shown is incorrect?

(a) 

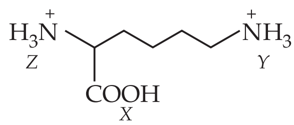
(b) 

(c) 

(d) 

70. For H_3PO_3 and H_3PO_4, the correct choice is
(a) H_3PO_3 is dibasic and reducing
(b) H_3PO_4 is dibasic and non-reducing
(c) H_3PO_4 is tribasic and reducing
(d) H_3PO_3 is tribasic and non-reducing.

71. The correct order of decreasing acid strengths of different groups in the given amino acid is



(a) $X > Z > Y$
(b) $Z > X > Y$
(c) $X > Y > Z$
(d) $Y > X > Z$

72. The equilibrium constants K_{p_1} and K_{p_2} for the reactions $X \rightleftharpoons 2Y$ and $Z \rightleftharpoons Q + R$, respectively are in the ratio of 1 : 9. If degree of dissociation of *X* and *Z* be equal then the ratio of total pressures at these equilibria is
(a) 1 : 9 (b) 1 : 36
(c) 1 : 1 (d) 1 : 3

73. Propan-1-ol can be prepared from propene by
(a) H_2O/H_2SO_4
(b) $Hg(OAc)_2/H_2O$ followed by $NaBH_4$
(c) B_2H_6 followed by H_2O_2
(d) CH_3CO_2H/H_2SO_4

74. What products are expected from the disproportionation reaction of hypochlorous acid?
(a) $HClO_3$ and Cl_2O (b) $HClO_2$ and $HClO_4$
(c) HCl and Cl_2O (d) HCl and $HClO_3$

75. What is the colour corresponding to the wavelength of light emitted when the electron in a hydrogen atom undergoes transition from $n = 4$ to $n = 2$?
(a) Blue (b) Red
(c) Yellow (d) Green

76. An optically active amine (*A*) of molecular formula $C_4H_{11}N$ is subjected to Hofmann's exhaustive methylation process and is followed by hydrolysis, an alkene (*B*) is produced which upon ozonolysis and subsequent hydrolysis yields formaldehyde and propanal. The amine (*A*) is
(a) $CH_3-\underset{\underset{NH_2}{|}}{CH}-CH_2CH_3$
(b) $CH_3-NH-\underset{\underset{C_2H_5}{|}}{CH}-CH_3$
(c) $CH_3-\underset{\underset{C_3H_7}{|}}{N}-CH_2CH_3$
(d) $CH_3CH_2CH_2CH_2-NH_2$

77. Which of the following statements is not correct?
(a) $La(OH)_3$ is less basic than $Lu(OH)_3$.
(b) In lanthanoid series, ionic radius of Ln^{3+} ions decreases.
(c) La is actually an element of transition series rather than lanthanoid series.
(d) Atomic radii of Zr and Hf are same because of lanthanoid contraction.

78. An organic compound *A* reacts with methyl magnesium iodide to form an addition product which on hydrolysis forms the compound *B*. Compound *B* gives blue colour salt in Victor Meyer's test. The compounds *A* and *B* respectively are
(a) acetaldehyde, *t*-butyl alcohol
(b) acetaldehyde, ethyl alcohol
(c) acetaldehyde, *iso*-propyl alcohol
(d) acetone, *iso*-propyl alcohol.

79. Which of the following statements is incorrect about peptide bond?
(a) C—N bond length in proteins is longer than usual bond length of C—N bond.
(b) Spectroscopic analysis shows planar structure of $-\underset{\underset{O}{||}}{C}-NH-$ bond.
(c) C—N bond length in proteins is smaller than usual bond length of C—N bond.
(d) None of these.

80. What is the effect of Frenkel defect on the density of ionic solids?
(a) The density of the crystal increases.
(b) The density of the crystal decreases.
(c) The density of the crystal remains unchanged.
(d) There is no relationship between density of a crystal and defect present in it.

MATHEMATICS

81. The equation of the tangent to the curve $y = \sqrt{9 - 2x^2}$ at the point where the ordinate and the abscissa are equal, is
(a) $2x + y - 3\sqrt{3} = 0$ (b) $2x + y + \sqrt{3} = 0$
(c) $2x + y - \sqrt{3} = 0$ (d) None of these

82. If $a_0, a_1, a_2, ..., a_n$ are real numbers such that

$$a_0 + \frac{a_1}{2} + \frac{a_2}{3} + ... + \frac{a_{n-1}}{n} + \frac{a}{n+1} = 0, (n \neq 1),$$

then the equation $a_0 + a_1x + a_2x^2 + ... + a_nx^n = 0$ has a real root lying between
(a) 0, 1 (b) 1, 2
(c) 1, 3 (d) None of these

83. The approximate value of $f(5.001)$, where

$f(x) = x^3 - 7x^2 + 15$, is
(a) –34.995 (b) –33.995
(c) –33.335 (d) –35.993

84. If $f(x) = (ab - b^2 - 2)x + \int_0^x (\cos^4\theta + \sin^4\theta)d\theta$ is decreasing function of *x* for all $x \in R$ and $b \in R$, *b* being independent of *x*, then
(a) $a \in (0, \sqrt{6})$ (b) $a \in (-\sqrt{6}, \sqrt{6})$
(c) $a \in (-\sqrt{6}, 0)$ (d) None of these

85. The minimum intercepts made by the axes on the tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ is
(a) 25 (b) 7
(c) 1 (d) None of these

86. A foot of the normal from the point (4, 3) to a circle is (2, 1) and a diameter of the circle has equation $2x - y = 2$. Then the equation of the circle is
(a) $x^2+y^2+2x-1=0$ (b) $x^2+y^2-2x-1=0$
(c) $x^2+y^2-2y-1=0$ (d) None of these

87. If the chord of contact of tangents from a point P to the parabola $y^2 = 4ax$, touches the parabola $x^2 = 4by$, then the locus of P is a/an
(a) circle (b) parabola
(c) ellipse (d) hyperbola

88. $\frac{x^2}{r^2-r-6}+\frac{y^2}{r^2-6r+5}=1$ will represent the ellipse, if r lies in the interval
(a) $(-\infty, 2)$ (b) $(3, \infty)$
(c) $(5, \infty)$ (d) $(1, \infty)$

89. The equation of the asymptotes of the hyperbola $2x^2 + 5xy + 2y^2 - 11x - 7y - 4 = 0$, are
(a) $2x^2+5xy+2y^2-11x-7y-5=0$
(b) $2x^2+4xy+2y^2-7x-11y+5=0$
(c) $2x^2+5xy+2y^2-11x-7y+5=0$
(d) None of the above

90. The derivative of $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ w.r.t. $\cos^{-1}\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}$ is
(a) 1 (b) –1
(c) $\frac{1}{2}$ (d) None of these

91. The probability of getting qualified in IIT/JEE and EAM/CET by a student are respectivley $\frac{1}{5}$ and $\frac{3}{5}$. The probability that the student gets qualified for at least one of these test, is
(a) $\frac{3}{25}$ (b) $\frac{8}{25}$
(c) $\frac{17}{25}$ (d) $\frac{22}{25}$

92. If the mean of a poisson distribution is $\frac{1}{2}$, then one ratio of $P(X = 3)$ to $P(X = 2)$ is
(a) 1 : 2 (b) 1 : 4
(c) 1 : 6 (d) 1 : 8

93. In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$. The probability that he copies is $\frac{1}{6}$ and the probability that his answer is correct given that he copied it is $\frac{1}{8}$. The probability that he knew the answer to the question given that he correctly answered it, is
(a) $\frac{24}{29}$ (b) $\frac{1}{4}$
(c) $\frac{3}{4}$ (d) $\frac{1}{2}$

94. Let S be a non-empty subset of R. Consider the following statement:
P : There is a rational number $x \in S$ such that $x > 0$.
Which of the following statements is the negation of the statement P ?
(a) There is a rational number $x \in S$ such that $x \le 0$.
(b) There is no rational number $x \in S$ such that $x \le 0$.
(c) Every rational number $x \in S$ satisfies $x \le 0$.
(d) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational.

95. Consider the following statements
P : Suman is brilliant
Q : Suman is rich
R : Suman is honest
The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as
(a) $\sim Q \leftrightarrow \sim P \wedge R$ (b) $\sim (P \wedge \sim R) \leftrightarrow Q$
(c) $\sim P \wedge (Q \leftrightarrow \sim R)$ (d) $\sim (Q \leftrightarrow (P \wedge \sim R))$

96. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, then $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2$ does not exceed
(a) 4 (b) 9
(c) 8 (d) 6

97. The diagonals of a parallelogram are given by $\vec{d}_1 = 2\hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{d}_2 = 3\hat{i} + 4\hat{j} - \hat{k}$ then area is

(a) $\frac{1}{2}\sqrt{50}$ sq. units (b) $\frac{1}{2}\sqrt{1005}$ sq. units

(c) $\frac{1}{2}\sqrt{1105}$ sq. units (d) None of these

98. Equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, on the plane $2x + 3y + 4z - 8 = 0$ be great circle, must be

(a) $x^2 + y^2 + z^2 + 4x + 2y - 6z + 10 = 0$

(b) $x^2 + y^2 + z^2 - 4x + 2y - 6z + 10 = 0$

(c) $x^2 + y^2 + z^2 - 6x - 4y - 2z - 10 = 0$

(d) $x^2 + y^2 + z^2 - 2x + 4y - 6z + 10 = 0$

99. The equations of the perpendicular from the origin to the $2x + 3y + 4z + 5 = 0$ and $x + 2y + 3z + 4 = 0$ must be

(a) $x + 2y - z = 0 = 3x - 2y - z$

(b) $2x + y + z = 0 = x - 2y - z$

(c) $x + 2y - z = 0 = 3x + 2y + z$

(d) $x - 2y + z = 0 = 3x + 2y + z$

100. Find the value of λ if the following equations are consistent

$x + y - 3 = 0$, $(1 + \lambda)x + (2 + \lambda)y - 8 = 0$,

$x - (1 + \lambda)y + (2 + \lambda) = 0$.

(a) $0, \frac{-1}{2}$ (b) $1, \frac{-5}{3}$

(c) $2, \frac{9}{5}$ (d) $4, -3$

101. Evaluate

$$\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \left(\frac{x^4}{1-x^4}\right)\cos^{-1}\left(\frac{2x}{1+x^2}\right)dx$$

(a) $\pi\left[\frac{-2}{\sqrt{3}} + \log\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{\pi}{6}\right]$

(b) $\frac{\pi}{3}\left[\frac{-2}{\sqrt{3}}\log\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) + \frac{\pi}{4}\right]$

(c) $\frac{\pi}{2}\left[\frac{-2}{\sqrt{3}} + \log\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{\pi}{6}\right]$

(d) $\pi\left[-2\log\left(\sqrt{3}-1\right) + \frac{\pi}{6}\right]$

102. Evaluate $\lim_{n\to\infty}\left[\frac{n!}{(kn)^n}\right]^{\frac{1}{n}}$, where k ($\neq$ 0) is a function and $n \in N$.

(a) $\frac{1}{k}e^{-1}$ (b) $-\frac{1}{k}e^{-1}$

(c) $k\,e^{-1}$ (d) $-k\,e$

103. If $f(x)$ be a continuous function such that $f(a - x) + f(x) = 0$ for all $x \in [0, a]$, then evaluate $\int_0^a \frac{dx}{1+e^{f(x)}}$.

(a) $-\frac{1}{2}a$ (b) $\frac{1}{2}a$

(c) $3a$ (d) $2a$

104. If A_1 is the area of the parabola $y^2 = 4ax$ lying between vertex and the latus rectum and A_2 is the area between the latus rectum and the double ordinate $x = 2a$, then $\frac{A_1}{A_2} =$

(a) $2\sqrt{2} - 1$ (b) $\frac{1}{7}(2\sqrt{2} + 1)$

(c) $\frac{1}{7}(2\sqrt{2} - 1)$ (d) none of these

105. Solution of the differential equation $\tan y \sec^2 x\, dx + \tan x \sec^2 y\, dy = 0$ is

(a) $\frac{\tan x}{\tan y} = K$ (b) $\tan x \tan y = K$

(c) $\tan x + \tan y = K$ (d) $\tan x - \tan y = K$

106. Which of the following functions is a solution of the differential equation?

$$\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0\,?$$

(a) $y = 2x^2 - 4$ (b) $y = 2x - 4$

(c) $y = 2x$ (d) $y = 2$

107. For two independent events A and B, $P(A \cap B) = \frac{3}{25}$, $P(A' \cap B) = \frac{8}{25}$, then $P(B) =$

(a) $\frac{3}{11}$ (b) $\frac{7}{25}$

(c) $\frac{11}{25}$ (d) none of these

108. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then

(a) $a > 0, b > 0$ (b) $a > 0, b < 0$

(c) $a < 0, b < 0$ (d) none of these

109. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ and $x = P^T Q^{2005} P$, then x is equal to

(a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{bmatrix}$

(c) $\frac{1}{4}\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$

(d) $\frac{1}{4}\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$

110. The factors of $\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix}$ are

(a) $x-a$, $x-b$ and $x+a+b$.
(b) $x+a$, $x+b$ and $x+a+b$.
(c) $x+a$, $x+b$ and $x-a-b$.
(d) $x-a$, $x-b$ and $x-a-b$.

111. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32} \cdot A_{32}$.

(a) 200 (b) 150
(c) 110 (d) 90

112. A balloon, which always remains spherical on inflation is being inflated by pumping in 900 cm^3/s of gas. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

(a) 11 cm/s (b) 2π cm/s
(c) $1/\pi$ cm/s (d) π^2 cm/s

113. Find the point on the curve $y = x^3 - 11x + 5$, at which the tangent is $y = x - 11$.

(a) (4, –7) (b) (0, 3)
(c) (–2, –13) (d) (2, –9)

114. Evaluate $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$.

(a) 0 (b) –2
(c) –1 (d) 2

115. Sketch the region lying in the first quadrant and bounded by $y = 9x^2$, $x = 0$, $y = 1$ and $y = 4$. Find the area of region using integration.

(a) $\frac{5}{3}$ sq. units (b) 10 sq. units
(c) $\frac{14}{9}$ sq. units (d) 9 sq. units

116. Solve the following differential equation

$$\log\left(\frac{dy}{dx}\right) = 3x + 4y$$

(a) $\frac{e^{-4y}}{-4} = e^{3x} + C$ (b) $\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$
(c) $e^{-4y} = e^{-3x} + C$ (d) $e^{4y} = e^{-3x} + C$

117. Let p be real and $|p| \geq 2$. If A, B and C are variable angles such that

$$\sqrt{p^2-4}\tan A + p\tan B + \sqrt{p^2+4}\tan C = 6p$$

then the minimum value of $\tan^2 A + \tan^2 B + \tan^2 C$ is

(a) 8 (b) 12
(c) 18 (d) 6

118. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{\alpha} = \vec{a} - (\vec{a}\cdot\vec{b})\vec{b}$ and $\vec{\beta} = \vec{a} \times \vec{b}$, then $|\vec{\beta}|$ is

(a) $|\vec{\alpha}|$ (b) $|\vec{\alpha}| + |\vec{\alpha}\cdot\vec{a}|$
(c) $|\vec{\alpha}| + |\vec{\alpha}\cdot\vec{b}|$ (d) $|\vec{\alpha}| + \vec{\alpha}\cdot(\vec{a}+\vec{b})$

119. The odds in favour of a book reviewed by three independent critics are, respectively, 5 : 2, 4 : 3 and 3 : 4. The probability that majority of the critics give favourable remark is

(a) $\frac{210}{343}$ (b) $\frac{209}{343}$
(c) $\frac{211}{343}$ (d) $\frac{205}{343}$

120. Bag A contains 5 white and 3 black balls. Bag B is empty. Four balls are taken at random from A and transferred to empty bag B. From B, a ball is drawn at random and is found to be black. Then, the probability that among the transferred balls three are black and one is white is

(a) $\frac{1}{8}$ (b) $\frac{7}{8}$
(c) $\frac{6}{7}$ (d) $\frac{1}{7}$

ANSWER KEY

1.	(d)	**2.**	(b)	**3.**	(b)	**4.**	(d)	**5.**	(a)	**6.**	(a)	**7.**	(c)	**8.**	(d)
9.	(d)	**10.**	(b)	**11.**	(d)	**12.**	(c)	**13.**	(c)	**14.**	(c)	**15.**	(c)	**16.**	(a)
17.	(b)	**18.**	(d)	**19.**	(d)	**20.**	(b)	**21.**	(c)	**22.**	(b)	**23.**	(d)	**24.**	(b)
25.	(d)	**26.**	(a)	**27.**	(b)	**28.**	(c)	**29.**	(c)	**30.**	(a)	**31.**	(b)	**32.**	(a)
33	(b)	**34.**	(b)	**35.**	(d)	**36.**	(a)	**37.**	(d)	**38.**	(a)	**39.**	(a)	**40.**	(b)
41.	(d)	**42.**	(b)	**43.**	(d)	**44.**	(c)	**45.**	(d)	**46.**	(c)	**47.**	(a)	**48.**	(b)
49.	(b)	**50.**	(b)	**51.**	(d)	**52.**	(b)	**53.**	(a)	**54.**	(c)	**55.**	(b)	**56.**	(d)
57.	(c)	**58.**	(c)	**59.**	(d)	**60.**	(b)	**61.**	(b)	**62.**	(d)	**63.**	(c)	**64.**	(c)
65.	(d)	**66.**	(a)	**67.**	(c)	**68.**	(b)	**69.**	(d)	**70.**	(a)	**71.**	(a)	**72.**	(b)
73.	(c)	**74.**	(d)	**75.**	(a)	**76.**	(a)	**77.**	(a)	**78.**	(c)	**79.**	(c)	**80.**	(c)
81.	(a)	**82.**	(a)	**83.**	(a)	**84.**	(b)	**85.**	(b)	**86.**	(b)	**87.**	(d)	**88.**	(c)
89.	(c)	**90.**	(a)	**91.**	(c)	**92.**	(c)	**93.**	(a)	**94.**	(c)	**95.**	(d)	**96.**	(b)
97.	(d)	**98.**	(d)	**99.**	(d)	**100.**	(b)	**101.**	(c)	**102.**	(a)	**103.**	(b)	**104.**	(b)
105.	(b)	**106.**	(b)	**107.**	(c)	**108.**	(b)	**109.**	(a)	**110.**	(a)	**111.**	(c)	**112.**	(c)
113.	(d)	**114.**	(a)	**115.**	(c)	**116.**	(b)	**117.**	(b)	**118.**	(a,c)	**119.**	(c)	**120.**	(d)

explanations

PHYSICS

1. **(d) :** Initially, $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$ ...(i)

when the third equal conductor touches B, the charge of B is shared equally between them.

$\therefore$ Charge on $B = \frac{q}{2}$ = charge on third conductor

Now this third conductor with charge $\left(\frac{q}{2}\right)$ touches C, their total charge $\left(q + \frac{q}{2}\right)$ is equally shared between them,

$\therefore$ Charge on $C = \frac{3q}{4}$ = charge on third conductor

$\therefore$ New force between B and C

$$F' = \frac{1}{4\pi\varepsilon_0 d^2}\left(\frac{q}{2}\times\frac{3q}{4}\right) = \frac{3}{8}F \quad \text{(using (i))}$$

2. **(b) :** $q_A = +5\times10^{-19}\ C$, $q_B = 3\times10^{-9}C$

Initial distance between charges (r_1)

$= 6\times\ 10^{-2}$ m

Final distance between charges $(r_2) = 5\times10^{-2}$ m

In moving the charge,

Work done = Final P.E. – Initial P.E.

$$= kq_1q_2\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

$$= 9\times10^9\times5\times10^{-9}\times3\times10^{-9} \times\left(\frac{1}{5\times10^{-2}}-\frac{1}{6\times10^{-2}}\right)$$

$$= 135\times10^{-9}\times\frac{1}{30\times10^{-2}} = 4.5\times10^{-7}\ \text{J}$$

3. **(b) :** Here, $\lambda_0 = 200$ nm, $\lambda = 100$ nm, $hc = 1240$ eV nm

Maximum kinetic energy $= \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$ (in eV)

$$= \frac{hc}{e}\left(\frac{1}{\lambda}-\frac{1}{\lambda_0}\right) = 1240\left(\frac{1}{100}-\frac{1}{200}\right) = 6.2\ \text{eV}$$

4. **(d) :** Energy of photon emitted,

$$E = 13.6\left(\frac{1}{1^2} - \frac{1}{5^2}\right) \text{eV} = 13.6 \times \frac{24}{25} \text{eV} = 13.06 \text{ eV}$$

Momentum of photon = $\frac{E}{c}$

The momentum of hydrogen atom is equal and opposite to the momentum of photon. If m is the mass of hydrogen atom (= 1.67 × 10^{-27} kg) and v is recoil speed of hydrogen atom, then

$$mv = \frac{E}{c}$$

$$v = \frac{E}{mc} = \frac{13.06 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27} \times 3 \times 10^8}$$

$v = 4.17 \text{ m s}^{-1} \approx 4 \text{ m s}^{-1}$

5. **(a) :** Given $D_{15} = 5.9 \times 10^{-3}$ m, $D_5 = 3.36 \times 10^{-3}$ m
$p = 10$ and $R = 1.0$ m
Wavelength of light,

$$\lambda = \frac{D^2_{(n+p)} - D^2_n}{4pR} = \frac{[(5.9)^2 - (3.36)^2] \times 10^{-6}}{4 \times 10 \times 1.0}$$

$\lambda = 5880 \text{ Å}$

6. **(a) :** Torque, $\vec{\tau} = \vec{p} \times \vec{E}$

∴ $\tau = pE\sin\theta$

or $4 = p \times 2 \times 10^5 \sin 30°$

or $p = \frac{4}{2 \times 10^5 \times \sin 30°} = 4 \times 10^{-5}$ Cm

Dipole moment, $p = ql$

∴ $q = \frac{p}{l} = \frac{4 \times 10^{-5}}{0.02} = 2 \times 10^{-3} \text{C} = 2 \text{mC}$

7. **(c) :** After charging, total charge on the capacitor

$q = CV$ (where $C = 10\ \mu$F)

∴ $q = 10 \times 10^{-6} \times 1000 = 10^{-2}$ C

When this charged capacitor is connected to uncharged capacitor then total charge remains same.

∴ $q = q_1 + q_2$

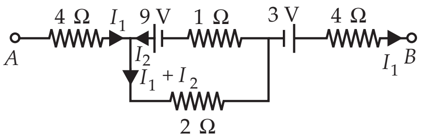
$10^{-2} = (C_1 + C_2)V$

∴ $V = \frac{10^{-2}}{16 \times 10^{-6}} = 625$ V

8. **(d)**

9. **(d) :** Radiation can be a hazard to living organisms because it produces ionization along its path of travel. This ionization can disrupt chemical bonds in essential macromolecules such as DNA and produce molecular fragments, which are free polyatomic ions that can interfere with enzyme action and other essential cell functions.

10. **(b) :**



$V_A - V_B = 16$ V

∴ $4I_1 + 2(I_1 + I_2) - 3 + 4I_1 = 16$ ...(i)

Using Kirchhoff's second law in the closed loop, we have

$9 - I_2 - 2(I_1 + I_2) = 0$ ...(ii)

Solving equations (i) and (ii), we get

$I_1 = 1.5$ A and $I_2 = 2$ A

∴ Current through 2 Ω resistor
= 2 + 1.5 = 3.5 A

11. **(d) :** Current in the circuit will be $I = \frac{(n-4)\varepsilon}{nr}$.

Hence, potential difference across A or B is

$$V = \varepsilon + Ir = \varepsilon + \frac{(n-4)\varepsilon}{nr} r = 2\varepsilon\left(1 - \frac{2}{n}\right)$$

12. **(c) :** Since dl and r for each element of the straight segments are either parallel or antiparallel.

Therefore

$\vec{dl} \times \vec{r} = 0$

Hence, B due to straight segment is also zero.

13. **(c) :** $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$

Here, $dl = \Delta x = 0.05$ m, $I = 10$ A, $r = 1$ m
and $\sin\theta = \sin 90° = 1$,

∴ $dB = 10^{-7} \times \frac{10 \times 0.05 \times 1}{(1)^2}$

$= 0.50 \times 10^{-7} = 5.0 \times 10^{-8}$ T

14. **(c) :** Suppose intensity of unpolarised light = 100.

∴ Intensity of polarised light from first nicol prism

$= \frac{I_0}{2} = \frac{1}{2} \times 100 = 50$

According to law of Malus,

$I = I_0\cos^2\theta = 50(\cos 60°)^2 = 50 \times \left(\frac{1}{2}\right)^2 = 12.5$

15. **(c) :** $B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r}$

Here, $I = \frac{2e}{t} = \frac{2 \times 1.6 \times 10^{-19}}{2} = 1.6 \times 10^{-19}$ A

∴ $B = \frac{\mu_0 I}{2r} = \frac{\mu_0 \times 1.6 \times 10^{-19}}{2 \times 0.8}$

$= \mu_0 \times 10^{-19}$ T

16. **(a) :** Magnetic dipole moment of circular loop is $m = NIA = 10 \times 0.5 \times 2 \times 10^{-4} = 10^{-3}$ A m^2

Magnetic field inside the solenoid carrying current

$B = \mu_0 n I = 4\pi \times 10^{-7} \times 10^3 \times 3 = 12\pi \times 10^{-4}$ T

Torque, $\tau = m\,B \sin\theta$

$= 10^{-3} \times 12\pi \times 10^{-4} \times \sin 90°$

$= 12\pi \times 10^{-7}$ N m

17. **(b) :** $x^2 + y^2 = 25$

$\therefore \quad r = 5$ m

$$\because \quad r = \frac{mv}{qB} \quad \text{or} \quad 5 = \frac{5 \times 10^{-3} \times 5}{5 \times 10^{-6} \times B}$$

$$\therefore \quad B = \frac{5 \times 10^{-3} \times 5}{5 \times 10^{-6} \times 5} = 10^3 \text{ T} = 1 \text{ kT}$$

The magnetic field will be 1 kT along z-axis.

18. **(d) :** $\lambda_{air} = \frac{1 \text{ mm}}{2000}$

$= 5 \times 10^{-4}$ mm $= 5 \times 10^{-7}$ m $= 5000$ Å

$$\lambda_{medium} = \frac{\lambda_{air}}{\mu} = \frac{5000\text{ Å}}{1.25} = 4000\text{ Å}$$

19. **(d) :** Here, $B = 2t$

$$\therefore \quad \frac{dB}{dt} = 2$$

Induced e.m.f., $|\varepsilon| = \frac{d\phi}{dt} = A\frac{dB}{dt} = 2\pi a^2$

Flow of charge per unit time through any section of the coil = induced current,

$$I = \frac{\varepsilon}{R} = \frac{2\pi a^2}{R} = \text{constant.}$$

Also, power generated, $P = I^2R$ = constant

Total charge passed through any section between $t = 0$ to $t = 2$ s is

$$q = It = \left(\frac{2\pi a^2}{R}\right)(2 - 0) = \frac{4\pi a^2}{R}$$

20. **(b) :** Normal to the plane of the coil will be perpendicular to the field $\vec{B}$.

$\therefore \quad \tau = IBA\sin 90° = IBA$

Area of equilateral triangle,

$$A = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times l \times l \sin 60° = \frac{\sqrt{3}}{4} l^2$$

$$\therefore \quad \tau = IB \times \frac{\sqrt{3}l^2}{4} \text{ or } l = 2\left(\frac{\tau}{\sqrt{3}BI}\right)^{1/2}$$

21. **(c) :** Magnifying power, $m = \frac{f_o}{f_e} = 9$ ...(i)

where f_o and f_e are the focal lengths of the objective and eyepiece respectively

Also, $f_o + f_e = 20$ cm ...(ii)

On solving (i) and (ii), we get

$f_o = 18$ cm, $f_e = 2$ cm

22. **(b) :** Magnetic potential at any point is the amount of work done in bringing a unit north pole from infinity to that point. At any point on the right bisector, the potentials due to the two poles are equal and opposite.

23. **(d) :** Impedance of the LR circuit

$$Z = \sqrt{R^2 + 4\pi^2\upsilon^2L^2}$$

$$= \sqrt{1^2 + 4\pi^2(50)^2 2^2} = \sqrt{394385} \approx 628$$

$$I = \frac{V}{Z} = \frac{5}{628} \text{ A}$$

Energy stored in the inductor,

$$U = \frac{1}{2}LI^2 = \frac{1}{2} \times 2 \times \left(\frac{5}{628}\right)^2 \text{ J} = 6.33 \times 10^{-5} \text{ J}$$

24. **(b) :** Here, $\frac{R_1}{R_2} = \frac{1}{2}$ or $R_2 = 2R_1$

The equivalent resistance of parallel combination

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{R_3} = \frac{3}{2R_1} + \frac{1}{R_3}$$

$$\text{or} \quad \frac{1}{R_3} = \frac{1}{1} - \frac{3}{2R_1} = 1 - \frac{3}{2R_1}$$

$$\text{or} \quad 1 = R_3 - \frac{3}{2}\frac{R_3}{R_1} \quad \text{or} \quad R_3 = 1 + \frac{3}{2}\frac{R_3}{R_1}$$

Since no resistance is in fraction, therefore minimum value of

$$\frac{R_3}{R_1} = \frac{2}{3}$$

$$\therefore \quad R_3 = 1 + \frac{3}{2} \times \frac{2}{3} = 2\,\Omega \text{ and } R_1 = 3\,\Omega$$

The maximum resistance value is

$R_2 = 2R_1 = 2 \times 3 = 6\,\Omega$

25. **(d) :** The bridge $ABCD$ is balanced if

$$\frac{10}{R_1} = \frac{30}{9} \quad \text{or} \quad R_1 = 3\,\Omega$$

When this bridge is balanced, no current flows in arm BD. Therefore, R_2 can have any finite value.

26. **(a) :** Let f_o and f_e be the focal lengths of the objective and eyepiece respectively. For normal adjustment, distance between the objective and the eyepiece (tube length) = $f_o + f_e$. Treating the line on the objective as the object, and the eyepiece as the lens, $u = -(f_o + f_e)$ and $f = f_e$.

$$\frac{1}{v} - \frac{1}{-(f_o + f_e)} = \frac{1}{f_e}$$

or $$\frac{1}{v} = \frac{1}{f_e} - \frac{1}{f_o + f_e} = \frac{f_o}{(f_o + f_e)f_e}$$

or $$v = \frac{(f_o + f_e)f_e}{f_o}.$$

$$\text{Magnification} = \left|\frac{v}{u}\right| = \frac{f_e}{f_o} = \frac{\text{image size}}{\text{object size}} = \frac{l}{L}.$$

$\therefore \quad \frac{f_o}{f_e} = \frac{L}{l}$ = magnification of telescope in normal adjustment.

27. **(b) :** Total initial resistance

$= G + R = 50\ \Omega + 2950\ \Omega = 3000\ \Omega$

$$\text{Current, } I = \frac{3\text{ V}}{3000\ \Omega} = 1 \times 10^{-3}\text{ A} = 1\text{ mA}$$

If the deflection has to be reduced to 20 divisions, then current

$$I' = \frac{1\text{ mA}}{30} \times 20 = \frac{2}{3}\text{ mA}$$

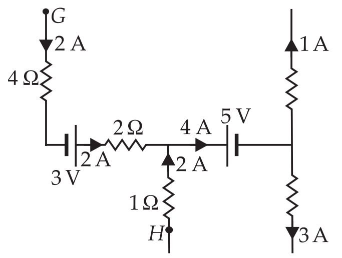
Let x be the effective resistance of the circuit, then

$$3\text{ V} = 3000\ \Omega \times 1\text{ mA} = x\ \Omega \times \frac{2}{3}\text{ mA}$$

or $$x = 3000 \times 1 \times \frac{3}{2} = 4500\ \Omega$$

$\therefore$ Resistance to be added $= (4500\ \Omega - 50\ \Omega)$
$= 4450\ \Omega$

28. **(c) :** The current distribution in a circuit is as shown in the figure.



Let V_G and V_H be the potentials at points G and H respectively.

$$\therefore \quad V_G - (2\text{ A})(4\ \Omega) + 3\text{ V} - (2\text{ A})(2\ \Omega) + (2\text{ A})(1\ \Omega) = V_H$$

$$V_G - 8\text{ V} + 3\text{ V} - 4\text{ V} + 2\text{ V} = V_H$$

$$V_G - V_H = 7\text{ V}$$

29. **(c) :** Here, $\varepsilon = 1.5$ V, $r = 2\ \Omega$, $R_{eq} = 5\ \Omega + 8\ \Omega$, $R = 5\ \Omega$

The potential difference across 5Ω resistance,

$$V = \left(\frac{\varepsilon}{R_{eq} + r}\right)R = \left[\frac{1.5\text{ V}}{(5\ \Omega + 8\ \Omega) + 2\ \Omega}\right]5\ \Omega = 0.5\text{ V}$$

30. **(a) :** Frequency of full wave rectifier
= 2 × input frequency = 2 × 50 = 100 Hz.

31. **(b) :** According to Gauss's law,
the electric flux through the sphere is

$$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{8.85 \times 10^{-13}\text{ C}}{8.85 \times 10^{-12}\text{ C}^2\text{ N}^{-1}\text{ m}^{-2}} = 0.1\text{ N C}^{-1}\text{ m}^2$$

32. **(a) :** In the usual setting of deflection magnetometer, field due to magnet (F) and horizontal component (H) of earth's field are perpendicular to each other. Therefore, the net field on the magnetic needle is

$$\sqrt{F^2 + H^2}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{I}{M\sqrt{F^2 + H^2}}} \quad \ldots\text{(i)}$$

When the magnet is removed,

$$T_0 = 2\pi\sqrt{\frac{I}{MH}} \quad \ldots\text{(ii)}$$

Also, $\frac{F}{H} = \tan\theta$

Dividing (i) by (ii), we get

$$\frac{T}{T_0} = \sqrt{\frac{H}{\sqrt{F^2 + H^2}}}$$

$$= \sqrt{\frac{H}{\sqrt{H^2\tan^2\theta + H^2}}} = \sqrt{\frac{H}{H\sqrt{\sec^2\theta}}} = \sqrt{\cos\theta}$$

$$\Rightarrow \quad \frac{T^2}{T_0^2} = \cos\theta \quad \therefore \quad T^2 = T_0^2\cos\theta$$

33. **(b) :** The most stable particle in the baryon group is proton.

34. **(b) :** The phenomenon of polarization shows that light has transverse nature.

35. **(d) :** Clearly, $2f = 20$ cm or $f = 10$ cm
Now, $u = -15$ cm, $f = 10$ cm

Using lens formula, $\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$

or $\frac{1}{v} + \frac{1}{15} = \frac{1}{10}$ or $\frac{1}{v} = \frac{1}{10} - \frac{1}{15}$

or $\frac{1}{v} = \frac{3-2}{30} = \frac{1}{30}$ or $v = 30$ cm

The change in image distance is (30 – 20) cm *i.e.*, 10 cm away from the lens.

36. **(a) :** As for minima,

$$n\lambda = a\sin\theta = \frac{ax}{f} \text{ or } \lambda = \frac{ax}{nf} \quad \left[\because \quad \sin\theta = \frac{x}{f}\right]$$

Here, $a = 0.3 \text{ mm} = 0.3 \times 10^{-3} \text{ m}$, $x = 5$ mm $= 5 \times 10^{-3}$ m, $n = 3$, $f = 1$ m.

$$\therefore \quad \lambda = \frac{0.3\times10^{-3}\times5\times10^{-3}}{3\times1} = 5\times10^{-7} \text{ m} = 5000 \text{ Å}.$$

37. **(d) :** From $\frac{1}{2}mv^2 = \frac{hc}{e\lambda} - \phi_0 (\text{in eV})$

$$\therefore \frac{1}{2}mv^2 = \frac{6.6\times10^{-34}\times3\times10^{8}}{2000\times10^{-10}\times1.6\times10^{-19}} - 4.2 = 6.2 - 4.2$$

$$= 2 \text{ eV} = 2 \times 1.6 \times 10^{-19} \text{ J}$$

$$\Rightarrow \quad v = \sqrt{\frac{2\times2\times1.6\times10^{-19}}{9.1\times10^{-31}}}$$

$$= \sqrt{\frac{6.4}{9.1}} \times 10^{6} \text{ m s}^{-1} = 0.84 \times 10^{6} \text{ m s}^{-1}.$$

$$= 8.4 \times 10^{5} \text{ m s}^{-1}$$

38. **(a) :** Here, $T_{1/2} = 4.47 \times 10^9$ yr, $N = \frac{60}{100}N_0$

$$\because \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{60}{100} = \left(\frac{1}{2}\right)^n \text{ or } 2^n = \frac{10}{6}$$

$\Rightarrow n \log 2 = \log 10 - \log 6 = 1 - 0.778 = 0.222$

$$\therefore \quad n = \frac{0.222}{\log 2} = \frac{0.222}{0.3} = 0.74$$

Now, $t = nT_{1/2} = 0.74 \times 4.47 \times 10^9$ yr
$= 3.3 \times 10^9$ yr.

39. **(a) :** Here, $A = 5°$, $\mu = 1.5$, $i = \theta$, $e = 0°$
As the emergent ray is normal to the refracting surface of the prism
Hence, for a small angled prism,
$\delta = (\mu - 1)A$,
$\delta = (1.5 - 1)5° = 2.5°$
Since, $A + \delta = i + e$,
$\Rightarrow 5° + 2.5° = \theta + 0°$
or $\theta = 7.5°$

40. **(b) :** As, $I = (e^{1000\,V/T} - 1)$ mA ...(i)
Here, $I = 5$ mA at $T = 300$ K
$dV = 0.01$ V
$\therefore \quad 5 = (e^{1000\,V/T} - 1) \Rightarrow e^{(1000\,V/T)} = 6$ mA.
Differentiating eqn. (i), we get

$$dI = \left(\frac{1000}{T}\right)e^{(1000\,V/T)}dV$$

$$= \frac{1000}{300}(6)(0.01) = 0.2 \text{ mA}.$$

CHEMISTRY

41. **(d) :** For a zero order reaction,
$A \longrightarrow P$

$$r = -\frac{d[A]}{dt} = k \quad \text{or} \quad -d[A] = kdt$$

When $t = 0$, $[A] = [A]_0$
At $t = t$, $[A] = [A]$

$$\text{Hence, } \int_{[A]_0}^{[A]} -d[A] = \int_0^t kdt$$

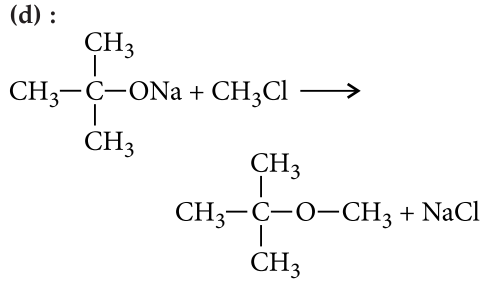
or $[A]_0 - [A] = kt$ or $[A] = [A]_0 - kt$
Thus, plot of $[A]$ *vs* t is linear with negative slope, k and intercept $[A]_0$.

42. **(b) :** Aldehydes reduce Benedict's solution.

43. **(d) :** Proline is a secondary amine.

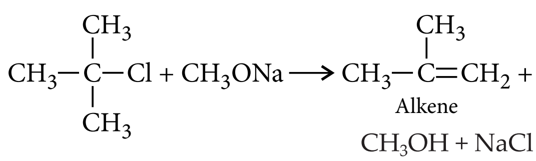
44. **(c) :** Positively charged ligands have suffix 'ium'.

45. **(d) :**



Methyl *t*-butyl ether

whereas,



Secondary and tertiary alkyl halides readily undergo elimination reaction rather than substitution.

46. **(c) :** For the reaction, $\frac{1}{2}A \rightarrow 2B$

$$\text{Rate} = -\frac{1}{1/2}\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$$

$$\Rightarrow \quad -\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$$

47. **(a) :** Nitrogen forms most basic compound with hydrogen among group 15 elements.

48. **(b) :** Ferric chloride, $FeCl_3$(*i.e.*, Fe^{3+}) cannot be further oxidised.

49. **(b) :** Packing efficiency of *ccp* or *fcc* = 74%
Packing efficiency of *bcc* = 68%
Packing efficiency of simple cubic = 52.4%

50. **(b) :** Boiling point of alcohols is higher than ethers due to H-bonding. In alcohols, the boiling point decreases with branching due to decrease in surface area. Hence, *n*-pentanol will have highest boiling point.

51. **(d) :** According to Debye Hückel-Onsager equation,

$$\lambda_C = \lambda_\infty - (B)\sqrt{C}$$

B is a constant depending upon the type of the electrolyte, the nature of the solvent and the temperature.

52. **(b) :** $KMnO_4$ in acidic medium oxidises 1° alcohol to acid.

$$RCH_2OH \xrightarrow{KMnO_4/H^+} RCOOH$$

53. **(a) :** Acidic strength of oxoacids of a particular halogen atom increases with increase in oxidation number thus, the order of acidic strength is

$$\overset{+1}{HClO} < \overset{+3}{HClO_2} < \overset{+5}{HClO_3} < \overset{+7}{HClO_4}\cdot$$

54. **(c) :** $A + B \rightarrow C$
Let rate = $k[A]^x\,[B]^y$
where order of reaction is $(x + y)$
Putting the values of exp. 1, 2, and 3, we get following equations:

$0.10 = k\,[0.012]^x\,[0.035]^y$ ...(i)
$0.80 = k\,[0.024]^x\,[0.070]^y$ ...(ii)
$0.10 = k\,[0.024]^x\,[0.035]^y$ ...(iii)

Dividing eq. (ii) by eq. (iii), we get

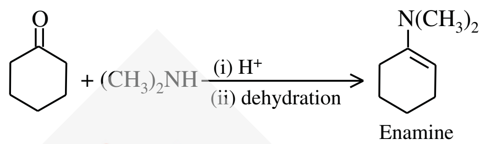
$$\frac{0.80}{0.10} = \left(\frac{0.070}{0.035}\right)^y \Rightarrow 2^y = 8 \Rightarrow y = 3$$

∴ Keeping [*A*] constant, [*B*] is doubled, rate becomes 8 times.
Dividing eq. (iii) by eq. (i), we get

$$\frac{0.10}{0.10} = \left(\frac{0.024}{0.012}\right)^x \Rightarrow 2^x = 1 \Rightarrow x = 0$$

∴ Keeping [*B*] constant, [*A*] is doubled, rate remains unaffected. Hence, rate is independent of [*A*].
Rate ∝ $[B]^3$

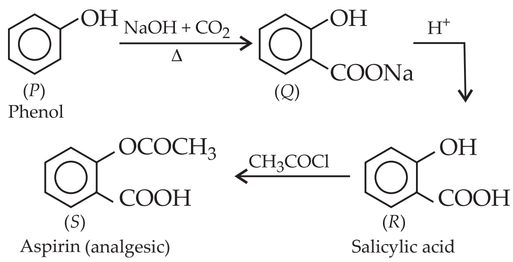
55. **(b) :**



56. **(d) :**

Element	%	Relative no. of atoms	Simple ratio
C	76.6	$\frac{76.6}{12} = 6.38$	6
H	6.38	$\frac{6.38}{1} = 6.38$	6
O	17.02	$\frac{17.02}{16} = 1.06$	1

∴ Empirical formula (*P*) = C_6H_6O
Empirical formula weight = 94
Molecular weight = 2 × *VD* = 2 × 47 = 94
∴ Molecular formula of *P* is C_6H_6O.
Since *P* gives colour with aq. $FeCl_3$ it has a phenolic group.
Compound *P* should be C_6H_5OH (phenol).



57. **(c)**

58. **(c) :** $Ag_2CrO_4 \rightleftharpoons 2Ag^+ + CrO_4^{2-}$
It is given that $[2Ag^+] = 1.5 \times 10^{-4}$ mol L^{-1}
∴ $[Ag^+] = 0.75 \times 10^{-4}$ mol L^{-1}
$[Ag^+] = [CrO_4^{2-}] = 0.75 \times 10^{-4}$ mol L^{-1}
$K_{sp} = [2Ag^+]^2\,[CrO_4^{2-}]$
$= (1.5 \times 10^{-4})^2\,(0.75 \times 10^{-4}) = 1.6875 \times 10^{-12}$

59. (d) : Corrosion is an electrochemical process in which Fe^{2+} are oxidised to Fe^{3+}.

Steps involved in the formation of rust are :

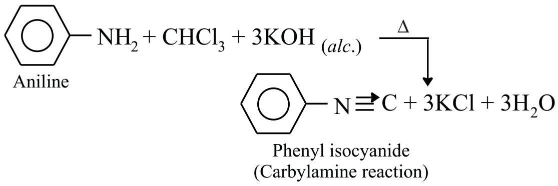
$Fe \rightarrow Fe^{2+} + 2e^-$ (Anode)

$O_2 + 4H^+ + 4e^- \rightarrow 2H_2O$ (Cathode)

$4Fe^{2+} + O_2 + 4H_2O \rightarrow 2Fe_2O_3 + 8H^+$

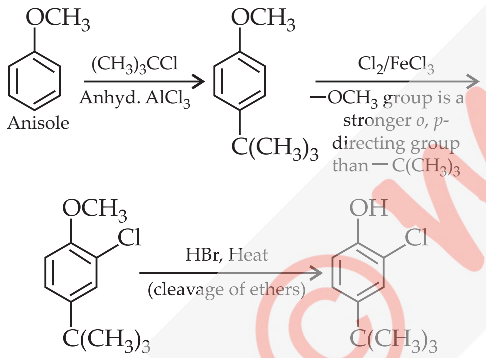
$Fe_2O_3 + xH_2O \rightarrow \underset{\text{(Rust)}}{Fe_2O_3 \cdot xH_2O}$

60. (b) :

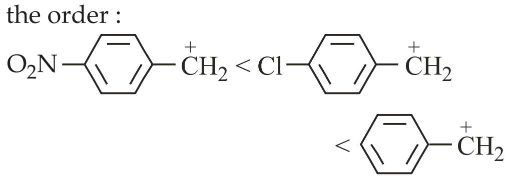


61. (b) : Due to lanthanoid contraction, atomic and ionic radii of Zr and Hf are almost equal.

62. (d) :



63. (c) : Electron withdrawing groups, *i.e.* $-NO_2$, –Cl etc. decreases the stability of carbocation. Since, the $-NO_2$ group is a stronger electron withdrawing group than –Cl, therefore, the stability of benzyl carbocations increases in the order :



Hence, the reactivity of benzyl alcohols increases in the same order, *i.e.*, II < III < I.

64. (c) : $[CrCl_2(H_2O)_4]Cl{\cdot}2H_2O$ has only one ionisable Cl out of three chlorine atoms present in the compound. Two Cl atoms which are non-ionisable are present in coordination sphere.

65. (d) : Dimethylamine is more basic than methylamine.

66. (a) :

	$2AB_{2(g)}$	$\rightleftharpoons$ $2AB_{(g)}$	$+\ B_{2(g)}$
Initially	1	0	0
At equilibrium	$(1-x)$	x	$x/2$

Total no. of moles at equilibrium

$$= (1-x) + x + \frac{x}{2} = \frac{2+x}{2}$$

Partial pressure = mole fraction × total pressure

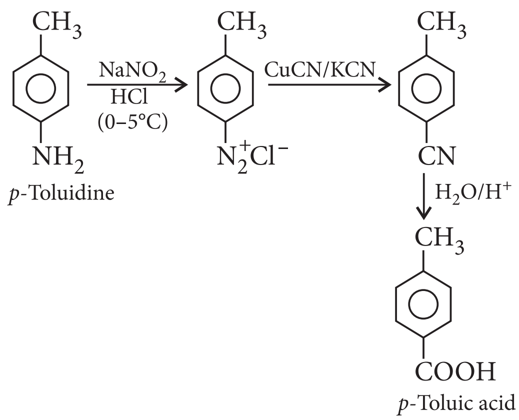
Applying $K_p = \dfrac{p_{AB}^2 \times p_{B_2}}{p_{AB_2}^2}$

$$= \frac{\left(\dfrac{\frac{x}{2+x}}{2} \times P\right)^2 \times \left(\dfrac{\frac{\frac{x}{2}}{2+x}}{2} \times P\right)}{\left(\dfrac{\frac{1-x}{2+x}}{2} \times P\right)^2} = \frac{Px^3}{(2+x)(1-x)^2}$$

Since $x << 1$ so $(1-x)^2$ can be neglected and $(2+x)$ can be taken as 2.

$$\therefore \quad K_p = \frac{Px^3}{2}$$

67. (c) :



68. (b) : $I_2 + H_2S \rightarrow 2HI + S$

$\Delta_r G^\circ = \Sigma\Delta_f G^\circ_{\text{Products}} - \Sigma\Delta_f G^\circ_{\text{Reactants}}$

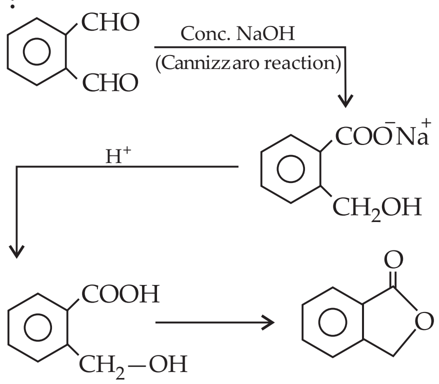
$\Delta_r G^\circ = (2 \times 1.8 + 0) - (0 + 33.8)$

$= -30.2$ kJ mol^{-1}

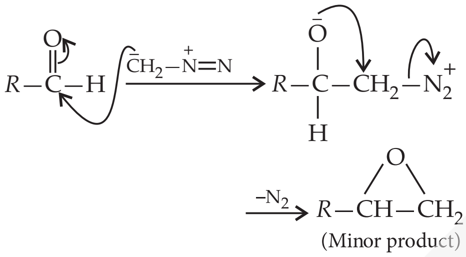
Hence, reaction is spontaneous in forward direction.

69. (d) :

(a)

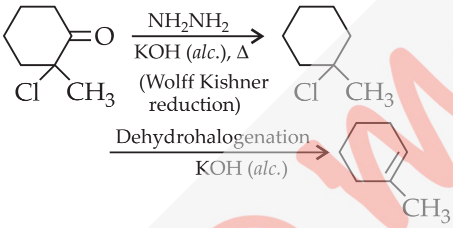


(b)



(c) is the Benzoin condensation.

(d) :



70. (a) :

H_3PO_3 $\quad$ H_3PO_4

Number of P — OH bonds determines the basicity of the acid and presence of P — H bond imparts reducing properties. Hence, H_3PO_3 is dibasic and reducing and H_3PO_4 is tribasic and non-reducing.

71. (a) : In a given amino acid, —COOH group is more acidic than $-\overset{+}{N}H_3$. Since —COOH has $-I$ effect which decreases with distance, therefore, effect is more pronounced on Z than on Y. As a result Z is more acidic than Y. Therefore, overall order of decreasing acid strengths is $X > Z > Y$.

72. (b) :

	$X \rightleftharpoons$	$2Y$;	$Z \rightleftharpoons$	Q +	R
Initial moles.	1	0	1	0	0
At equilibrium	$1-\alpha$	2α	$1-\alpha$	α	α

$$K_{p_1} = \frac{p_Y^2}{p_X} = \frac{\left(\frac{2\alpha}{1+\alpha}P_1\right)^2}{\left(\frac{1-\alpha}{1+\alpha}P_1\right)}$$

$$\Rightarrow K_{p_1} = \frac{4\alpha^2 P_1}{1-\alpha^2} \quad \text{... (i)}$$

$$K_{p_2} = \frac{p_Q\, p_R}{p_Z} = \frac{\left(\frac{\alpha}{1+\alpha}p_2\right)\left(\frac{\alpha}{1+\alpha}p_2\right)}{\left(\frac{1-\alpha}{1+\alpha}p_2\right)}$$

$$\Rightarrow K_{p_2} = \frac{\alpha^2 P_2}{1-\alpha^2} \quad \text{...(ii)}$$

Given is $\frac{K_{p_1}}{K_{p_2}} = \frac{1}{9}$ ...(iii)

Substituting values of equation (i) and (ii) into (iii), we get

$$\frac{\frac{4\alpha^2 P_1}{1-\alpha^2}}{\frac{\alpha^2 P_2}{1-\alpha^2}} = \frac{1}{9} \Rightarrow \frac{4P_1}{P_2} = \frac{1}{9} \Rightarrow \frac{P_1}{P_2} = \frac{1}{36}$$

73. (c) : Hydroboration-oxidation gives anti-Markownikoff's product.

$$\underset{\text{Propene}}{CH_3-CH=CH_2} \xrightarrow[\text{(ii) } H_2O_2/OH^-]{\text{(i) } B_2H_6} \underset{\text{Propan-1-ol}}{CH_3-CH_2-CH_2OH}$$

74. (d) : $3HClO_{(aq)} \rightarrow HClO_{3(aq)} + 2HCl_{(aq)}$

It is a disproportionation reaction of hypochlorous acid where the oxidation number of Cl changes from +1 (in ClO^-) to +5 (in ClO_3^-) and –1 (in Cl^-).

75. (a) :

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = 109677\left[\frac{1}{2^2} - \frac{1}{4^2}\right] \text{cm}^{-1}$$

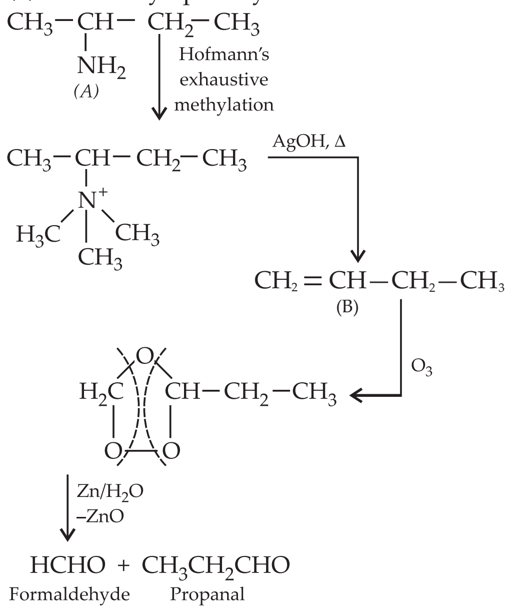
$$= 20564.4 \text{ cm}^{-1}$$

$$\lambda = \frac{1}{20564.4 \text{ cm}^{-1}} = 486 \times 10^{-7} \text{cm}$$

or 486×10^{-9} m = 486 nm

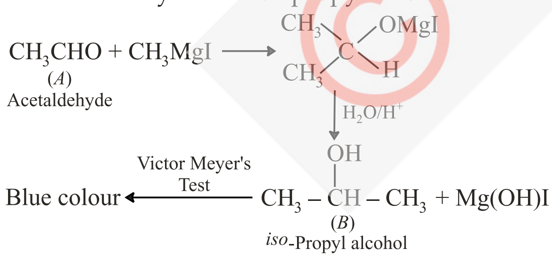
Colour corresponding to this wavelength is blue.

76. **(a)** : The only optically active molecule is

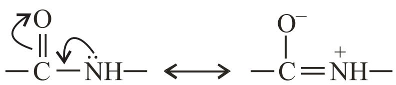


77. **(a)** : $La(OH)_3$ is more basic than $Lu(OH)_3$.

78. **(c)** : In Victor Meyer's test, blue colour salt is given by secondary alcohols. It means compound *B* must be a secondary alcohol. Only aldehydes with Grignard reagent yield secondary alcohols. Aldehyde should have one carbon less than the secondary alcohol since Grignard reagent contains methyl group. These conditions are satisfied by acetaldehyde and *iso*-propyl alcohol.



79. **(c)** : Due to resonance, C—N bond acquires some double bond character.



As a result C—N bond length in proteins becomes smaller than usual C—N bond length.

80. **(c)** : In Frenkel defect, ions get displaced from their original position and move to interstitial sites. Hence, there is no change in the density of the crystal.

MATHEMATICS

81 **(a)** : Given, $x_1 = y_1$

$\therefore \quad x_1 = \sqrt{9 - 2x_1^2}$

$\Rightarrow \quad x_1^2 = 9 - 2x_1^2 \quad \Rightarrow \quad x_1 = \pm\sqrt{3}.$

Since, $y_1 > 0$, therefore, the point is $(\sqrt{3}, \sqrt{3})$.

Now, $y = \sqrt{9 - 2x^2}$

On differentiating *w.r.t.x*, we get

$$2y\frac{dy}{dx} = -4x \Rightarrow \frac{dy}{dx} = -\frac{2x}{y}$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{(\sqrt{3}, \sqrt{3})} = -2$$

So, the required equation of tangent is

$$(y - \sqrt{3}) = -2(x - \sqrt{3})$$

$$\Rightarrow \quad 2x + y - 3\sqrt{3} = 0$$

82. **(a)** : Consider the polynomial

$$f(x) = a_0 x + \frac{a_1 x^2}{2} + \frac{a_2 x^3}{3} + \ldots + a_n \frac{x^{n+1}}{n+1} \qquad \text{...(i)}$$

We have, $f(0) = 0$

and $f(1) = a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \ldots + \frac{a_n}{n+1} = 0$

Therefore, 0 and 1 are roots of $f(x)$.

Hence, by algebraic interpretation of Rolle's theorem,

$f'(x) = 0$

i.e., $a_0 + a_1 x + a_2 x^2 + \ldots. + a_n x^n = 0$ must have a root lying between 0 and 1.

83. **(a)** : Firstly, break the number 5.001 as $x = 5$ and $\Delta x = 0.001$ and use the relation $f(x + \Delta x) \simeq f(x) + \Delta x f'(x)$.

Consider $f(x) = x^3 - 7x^2 + 15$

$\Rightarrow \quad f'(x) = 3x^2 - 14x$

Let $x = 5$ and $\Delta x = 0.001$

Also, $f(x + \Delta x) \simeq f(x) + \Delta x f'(x)$

Therefore,

$$f(x + \Delta x) \simeq (x^3 - 7x^2 + 15) + \Delta x(3x^2 - 14x)$$

$$\Rightarrow \quad f(5.001) \simeq (5^3 - 7 \times 5^2 + 15) + (3 \times 5^2 - 14 \times 5)(0.001)$$

(as $x = 5$, $\Delta x = 0.001$)

$= 125 - 175 + 15 + (75 - 70)(0.001)$

$= -35 + (5)(0.001) = -35 + 0.005 = -34.995$

84. **(b)** : $f'(x) = (ab - b^2 - 2) + \cos^4 x + \sin^4 x < 0$

$= ab - b^2 - 2 + (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x < 0$

$$\Rightarrow \quad ab - b^2 - 1 < \left(\frac{1}{2}\right)\sin^2 2x < \frac{1}{2}$$

$\Rightarrow \quad 2ab - 2b^2 - 2 < 1$

$\Rightarrow \quad 2b^2 - 2ab + 3 > 0$

$\therefore \quad (-2a)^2 - 4.2.3 < 0 \qquad [\because b^2 - 4ac < 0]$

$\Rightarrow \quad a^2 < 6 \Rightarrow -\sqrt{6} < a < \sqrt{6} \Rightarrow a \in (-\sqrt{6}, \sqrt{6})$

85. **(b) :** Any tangent to the ellipse is

$\frac{x}{4}\cos t + \frac{y}{3}\sin t = 1$, where the point of contact is $(4\cos t, 3\sin t)$

or $\quad \frac{x}{4\sec t} + \frac{y}{3\operatorname{cosec} t} = 1$

It means the axes Q (4 sec$t$, 0) and R(0, 3cosec$t$).

$\therefore$ The distance of the line segment QR is $QR^2 = D = 16\sec^2 t + 9\operatorname{cosec}^2 t$

So, the minimum value of D is $(4 + 3)^2$ or $QR = 7$.

86. **(b) :** The line joining (4, 3) and (2, 1) is also along a diameter. So, the centre of the circle is the intersection of the diameters $2x - y = 2$

and $y - 3 = \frac{3-1}{4-2}(x - 4)$.

On solving these two equations, the coordinates of centre of the circle are (1, 0).

Also, the radius of circle = the distance between (1, 0) and (2, 1) $= \sqrt{2}$.

Equation of circle is $(x - 1)^2 + y^2 = 2$

$\Rightarrow x^2 + y^2 - 2x - 1 = 0$

87. **(d) :** Let $P(h, k)$ be a point. Then, the chord of contact of tangents from P to $y^2 = 4ax$ is

$$ky = 2a(x + h) \qquad \text{...(i)}$$

This touches the parabola $x^2 = 4by$.

So, it should be of the form

$$x = my + \frac{b}{m} \qquad \text{...(ii)}$$

Eq. (i) can be rewritten as

$$x = \left(\frac{k}{2a}\right)y - h \qquad \text{...(iii)}$$

Since, Eqs. (ii) and (iii) represent the same line.

$\therefore \quad m = \frac{k}{2a}$

and $\quad \frac{b}{m} = -h$

Eliminating m from these two equations, we get

$2ab = -hk$

Hence, the locus of $P(h, k)$ is $xy = -2ab$, which is a hyperbola.

88. **(c) :** $r^2 - r - 6 > 0$ and $r^2 - 6r + 5 > 0$

$\Rightarrow \quad (r - 3)(r + 2) > 0$ and $(r - 1)(r - 5) > 0$

$\Rightarrow \quad \{r < -2 \text{ or } r > 3\} \cap \{r < 1 \text{ or } r > 5\}$

$\Rightarrow \quad r < -2$ or $r > 5$

89. **(c) :** Equation of hyperbola is

$$2x^2 + 5xy + 2y^2 - 11x - 7y - 4 = 0 \qquad \text{...(i)}$$

Let asymptotes of the hyperbola (i) be

$$2x^2 + 5xy + 2y^2 - 11x - 7y + c = 0 \qquad \text{...(ii)}$$

Since, Eq (ii) represents a pair of lines

$$\therefore \begin{vmatrix} 2 & 5/2 & -11/2 \\ 5/2 & 2 & -7/2 \\ -11/2 & -7/2 & c \end{vmatrix} = 0$$

$$\Rightarrow -\frac{11}{2}\left(-\frac{35}{4} + 11\right) + \frac{7}{2}\left(-7 + \frac{55}{4}\right) + c\left(4 - \frac{25}{4}\right) = 0$$

$$\Rightarrow \frac{9}{4}c = -\frac{99}{8} + \frac{189}{8} = \frac{90}{8} \Rightarrow c = 5$$

$\therefore$ Equation of the required asymptotes is $2x^2 + 5xy + 2y^2 - 11x - 7y + 5 = 0$

90. **(a) :** Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right)$

and $t = \cos^{-1}\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}$

Put $x = \tan\theta$

$$\therefore \quad y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$$

and $\quad t = \cos^{-1}\sqrt{\frac{1+\sec\theta}{2\sec\theta}} = \cos^{-1}\sqrt{\frac{1+\cos\theta}{2}}$

$$= \cos^{-1}\left(\cos\frac{\theta}{2}\right) = \frac{\theta}{2}$$

$$\therefore \quad y = \frac{\theta}{2} = t \Rightarrow \frac{dy}{dt} = 1$$

91. **(c)** : We have $P(A)=\frac{1}{5}, P(B)=\frac{3}{5}$

Required probability

$=P(A)P(\bar{B})+P(\bar{A})P(B)+P(A)P(B)$

$=\frac{1}{5}\left(1-\frac{3}{5}\right)+\left(1-\frac{1}{5}\right)\cdot\frac{3}{5}+\frac{1}{5}\cdot\frac{3}{5}=\frac{17}{25}$

92. **(c)** : We have, $m=\frac{1}{2}$

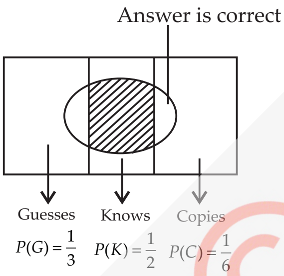
As $P(X=r)=\frac{e^{-m}m^r}{r!}$

$$\therefore \quad \frac{P(X=3)}{P(X=2)}=\frac{e^{-1/2}\left(\frac{1}{2}\right)^3}{3!}\div\frac{e^{-1/2}\left(\frac{1}{2}\right)^2}{2!}$$

$$=\frac{e^{-\frac{1}{2}}\left(\frac{1}{2}\right)^3}{3!}\times\frac{2!}{e^{-\frac{1}{2}}\left(\frac{1}{2}\right)^2}$$

$=1:6$

93. **(a)** :



$$P\left(\frac{\text{Knows}}{\text{Answer correct}}\right)$$

$$=\frac{P(\text{Knows})\times P\left(\frac{\text{Answer correct}}{\text{Knows}}\right)}{P(\text{Guesses})\times P\left(\frac{\text{Answer correct}}{\text{Guesses}}\right)+P(\text{Knows})\times P\left(\frac{\text{Answer correct}}{\text{Knows}}\right)+P(\text{Copies})\times P\left(\frac{\text{Answer correct}}{\text{Copies}}\right)}$$

$$=\frac{\frac{1}{2}\times 1}{\frac{1}{3}\times\frac{1}{4}+\frac{1}{2}\times 1+\frac{1}{6}\times\frac{1}{8}}=\frac{24}{29}$$

94 **(c)** : The given statement is

P : at least one rational $x \in S$ such that $x > 0$.

The negation would be : There is no rational number $x \in S$ such that $x > 0$

which is equivalent to all rational numbers $x \in S$ satisfy $x \leq 0$.

95. **(d)** : The statement can be written as

$P \wedge \sim R \Leftrightarrow Q$

Thus the negation is

$\sim (Q \leftrightarrow P \wedge \sim R)$

96. **(b)** : Given expression

$6-2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$

As $\quad |\vec{a}+\vec{b}+\vec{c}|\geq 0$

$\Rightarrow \quad 3+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})\geq 0$

$\Rightarrow \quad -3-2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})\leq 0$

$\Rightarrow \quad 6-2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})\leq 9$

97. **(d)** : $\frac{1}{2}|\vec{d}_1\times\vec{d}_2|=\frac{1}{2}\sqrt{698}$

$\Rightarrow$ (d) is correct.

98. **(d)** : Let the required sphere

$x^2+y^2+z^2+7y-2z+2+\lambda(2x+3y+4z-8)=0$

whose centre must lie on the plane

$2x+3y+4z-8=0$

Hence centre : $\left(-\lambda, \frac{1}{2}(-7-3\lambda), (1-2\lambda)\right)$

$\Rightarrow \quad 2(-\lambda)+\frac{3}{2}(-7-3\lambda)+4(1-2\lambda)=8$

$\Rightarrow \quad \lambda=-1$

Hence, $x^2+y^2+z^2-2x+4y-6z+10=0$

99. **(d)** : The plane passing through the origin and containing the given planes is

$(2x+3y+4z+5)+k(x+2y+3z+4)=0$ and

The value of k can be obtained by putting $x=y=z=0$

We get $k=-\frac{5}{4}\Rightarrow 3x+2y+z=0$

Let P is any point on the line of intersection of given plane.

Let D.R's of the line of intersection l, m, n which are related to the D.R's of normals of the planes.

$\Rightarrow \quad 2l+3m+4n=0$ and $l+2m+3n=0$

$\Rightarrow \quad \frac{l}{1}=\frac{m}{-2}=\frac{n}{1}$

Hence, $1(x-0)-2(y-0)+1(z-0)=0$

i.e., $x-2y+z=0$

So, $x-2y+z=0=3x+2y+z$ is correct.

100. (b) : Here, there are 3 equations in 2 unknowns. The equations are consistent if $\Delta = 0$

i.e., if $\begin{vmatrix} 1 & 1 & -3 \\ 1+\lambda & 2+\lambda & -8 \\ 1 & -1-\lambda & 2+\lambda \end{vmatrix} = 0$

(Operating $C_2 - C_1$, $C_3 + 3C_1$)

if $\begin{vmatrix} 1 & 0 & 0 \\ 1+\lambda & 1 & -5+3\lambda \\ 1 & -2-\lambda & 5+\lambda \end{vmatrix} = 0$

i.e., if $(5+\lambda)+(2+\lambda)(-5+3\lambda)=0$

i.e., if $3\lambda^2+2\lambda-5=0$ *i.e.*, if $\lambda = 1, \frac{-5}{3}$

101. (c) : $I = \int_{-1/\sqrt{3}}^{1/\sqrt{3}} \left(\frac{x^4}{1-x^4}\right)\left(\frac{\pi}{2} - 2\tan^{-1}x\right)dx$

$= \frac{\pi}{2}\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4}dx - 2\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \left(\frac{x^4}{1-x^4}\right)\tan^{-1}x\,dx$

$= 2\cdot\frac{\pi}{2}\int_0^{1/\sqrt{3}} \frac{x^4}{1-x^4}dx - 0$

[$\because$ first integrand is even function and second integrand is an odd function of x]

$= \pi\int_0^{1/\sqrt{3}} -1 + \frac{\frac{1}{2}}{1-x^2} + \frac{\frac{1}{2}}{1+x^2}dx$

$= \pi\left[-x + \frac{1}{2}\log_e\left|\frac{1+x}{1-x}\right| + \frac{1}{2}\tan^{-1}x\right]_0^{\frac{1}{\sqrt{3}}}$

$= \frac{\pi}{2}\left[-\frac{2}{\sqrt{3}} + \log\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{\pi}{6}\right]$

102. (a) : $\lim_{n\to\infty}\left[\frac{n!}{k^n n^n}\right]^{\frac{1}{n}}$

$= \lim_{n\to\infty}\frac{1}{k}\left(\frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\ldots\frac{n}{n}\right)^{\frac{1}{n}}$

$= \frac{1}{k}e^{\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(\frac{r}{n}\right)}$

$= \frac{1}{k}\cdot e^{\int_0^1 \log x\,dx} = \frac{1}{k}e^{[x\log x]_0^1 - \int_0^1 x\cdot\frac{1}{x}dx}$

$= \frac{1}{k}e^{-1}\left[\because \lim_{x\to 0}(x\log x) = \lim_{x\to 0}\left(\frac{\log x}{\frac{1}{x}}\right)\left(\frac{\infty}{\infty}\right) = \lim_{x\to 0}\frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0}(-x) = 0\right]$

103. (b) : $I = \int_0^a \frac{dx}{1+e^{f(x)}} = \int_0^a \frac{dx}{1+e^{f(a-x)}}$

$= \int_0^a \frac{dx}{1+e^{-f(x)}}$ (by given condition)

$\Rightarrow I = \int_0^a \frac{e^{f(x)}}{e^{f(x)}+1}dx$

$\therefore$ Adding, we get

$2I = \int_0^a \frac{1+e^{f(x)}}{1+e^{f(x)}}dx = \int_0^a 1dx = a$

$\Rightarrow I = \frac{1}{2}a.$

104. (b) : $A_1 = 2\int_0^a \sqrt{4ax}\,dx = \frac{8a^2}{3}.$

$A_2 = 2\int_0^{2a}\sqrt{4ax} - \int_0^a \sqrt{4ax}\,dx = \frac{16}{3}\sqrt{2}a^2 - \frac{8a^2}{3}$

$\therefore \frac{A_1}{A_2} = \frac{1}{2\sqrt{2}-1} = \frac{2\sqrt{2}+1}{7}.$

105. (b) : Given differential equation is

$\frac{\sec^2 x}{\tan x}dx + \frac{\sec^2 y\,dy}{\tan y} = 0$

Integrating, $\log|\tan x| + \log|\tan y| = C$

$\Rightarrow \tan x\tan y = \pm e^C = K.$

106 (b) : Clearly, $y = 2x-4$ gives $\frac{dy}{dx} = 2$ and these two equations together satisfy the given differential equation.

($\because 2^2 - x(2) + 2x - 4 = 0$ is true for all x)

107. (c) : Since A and B are independent,

$P(A\cap B) = P(A)P(B)$ and

$P(A'\cap B) = P(A')P(B)$

$\Rightarrow P(A)P(B) = \frac{3}{25}$ and $(1-P(A))P(B) = \frac{8}{25}$

$\Rightarrow P(B) = \frac{11}{25}.$

108. (b) : $y = \frac{1}{x} \Rightarrow \frac{dy}{dx} = -\frac{1}{x^2}$

$\therefore$ Slope of normal is x^2.

Slope of line $ax + by + c = 0$ is $-\frac{a}{b}$

$\therefore \quad x^2 = -\frac{a}{b} \Rightarrow -\frac{a}{b} > 0 \Rightarrow \frac{a}{b} < 0$

$\therefore$ a and b have opposite signs.

109. (a) : $P^T P = I$

$Q = PAP^T$ so that

$x = P^T Q^{2005} P = P^T (PAP^T)^{2005} P$

$= P^T PAP^T (PAP^T)^{2004} P$

$$= A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}.$$

110. (a) : Operate $R_2 - R_1$ and $R_3 - R_1$, we get

$$\begin{vmatrix} x & a & b \\ a-x & x-a & 0 \\ a-x & b-a & x-b \end{vmatrix}$$

$$= (x-a)\begin{vmatrix} x & a & b \\ -1 & 1 & 0 \\ a-x & b-a & x-b \end{vmatrix}$$

Operate $C_1 + C_2 + C_3$

$$= (x-a)\begin{vmatrix} x+a+b & a & b \\ 0 & 1 & 0 \\ 0 & b-a & x-b \end{vmatrix}$$

$= (x-a)(x+a+b)(x-b)$.

111. (c) : Let $A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Here, $a_{32} = 5$

Given that, A_{ij} is the cofactor of the element a_{ij} of A. Then,

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = (-1)^5(8-30) = -(-22) = 22$$

$\therefore \quad a_{32} \cdot A_{32} = 5 \times 22 = 110$

112. (c) : Let r be the radius and V be volume of the balloon.

Given, $r = 15$ cm, $\frac{dV}{dt} = 900 \text{ cm}^3/\text{s}$

Volume of the balloon (sphere) is $V = \frac{4}{3}\pi r^3$

On differentiating both sides w.r.t. t, we get

$$\frac{dV}{dt} = \frac{4}{3}\pi \times 3r^2 \times \frac{dr}{dt}$$

$$\Rightarrow \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

On substituting the given values, we get

$$900 = 4\pi \times (15^2) \times \frac{dr}{dt}$$

$$\therefore \quad \frac{dr}{dt} = \frac{900}{4\pi \times 225} = \frac{1}{\pi} \text{cm/s}$$

Hence, the radius of the balloon is increasing at the rate of $\frac{1}{\pi}$ cm/s, when the radius is 15 cm.

113. (d) : Given curve is

$y = x^3 - 11x + 5$ ...(i)

and given line is

$y = x - 11$

$\Rightarrow \quad x - y - 11 = 0$ ...(ii)

On differentiating Eq. (i) both sides w.r.t. x, we get

$$\frac{dy}{dx} = 3x^2 - 11$$

$\therefore$ Slope of tangent, $m_1 = 3x^2 - 11$

From Eq. (ii),

Slope of tangent, $m_2 = 1$

$$\left(\because \; m = -\frac{\text{coefficient of } x}{\text{coefficient of } y}\right)$$

Since, line of Eq. (ii) is the tangent to the curve.

$\therefore \quad m_1 = m_2 \Rightarrow 3x^2 - 11 = 1$

($\because$ slope of the curve = slope of the tangent)

$\Rightarrow \quad 3x^2 = 12 \Rightarrow x^2 = 4$

$\therefore \quad x = \pm 2$

When $x = 2$

Then, $y = (2)^3 - 11(2) + 5 = 8 - 22 + 5 = -9$

When $x = -2$

Then, $y = (-2)^3 - 11(-2) + 5 = -8 + 22 + 5 = 19$

$\therefore$ Points are $(2, -9)$ and $(-2, 19)$.

The point $(2, -9)$ lies on the line, $y = x - 11$ but the point $(-2, 19)$ does not lie on the line $y = x - 11$.

Hence, the required point is $(2, -9)$.

114. (a) : Let $I=\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$

$$=\int_0^{\pi/2}\log(4+3\sin x)dx-\int_0^{\pi/2}\log(4+3\cos x)dx$$

$$\left[\because \log\frac{m}{n}=\log m-\log n\right]$$

$$=\int_0^{\pi/2}\log\left[4+3\sin\left(\frac{\pi}{2}-x\right)\right]dx-\int_0^{\pi/2}\log(4+3\cos x)dx$$

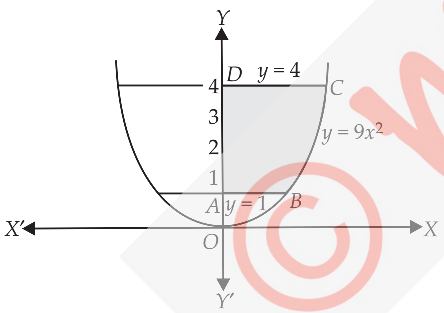
$$\left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

$$=\int_0^{\pi/2}\log(4+3\cos x)dx-\int_0^{\pi/2}\log(4+3\cos x)dx$$

$\therefore\quad I=0$

115. (c) : Given curve is $y=9x^2$

$\Rightarrow\quad x^2=\frac{1}{9}y\Rightarrow x=\frac{1}{3}\sqrt{y}$ ...(i)



It is a parabolic curve, which opens upwards. And it is symmetrical about Y-axis and passes through the origin.

$\therefore$ Required area of bounded region $ABCDA$ is

$$=\int_1^4 x\,dy=\frac{1}{3}\int_1^4\sqrt{y}\,dy \qquad \text{[from Eq. (i)]}$$

$$=\frac{1}{3}\left[\frac{y^{3/2}}{3/2}\right]_1^4=\frac{1}{3}\times\frac{2}{3}[(4)^{3/2}-(1)^{3/2}]$$

$$=\frac{2}{9}[(2)^3-1]=\frac{14}{9}\text{ sq units}$$

Therefore, the required area is $\frac{14}{9}$ sq units.

116. (b) : Given differential equation is

$$\log\left(\frac{dy}{dx}\right)=3x+4y$$

$$\therefore\quad \frac{dy}{dx}=e^{3x+4y}$$

$$\Rightarrow\quad \frac{dy}{dx}=e^{3x}\cdot e^{4y}$$

On separating the variables, we get

$$\frac{dy}{e^{4y}}=e^{3x}dx$$

On integrating both sides, we get

$$\int e^{-4y}dy=\int e^{3x}dx$$

$$\Rightarrow\quad \frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}+C\quad\left(\because\ \int e^{ax}dx=\frac{e^{ax}}{a}\right)$$

which is the required solution.

117. (b) : Consider the vectors

$$\vec a=\sqrt{p^2-4}\,\hat i+p\hat j+\sqrt{p^2+4}\,\hat k$$

and

$$\vec b=(\tan A)\hat i+(\tan B)\hat j+(\tan C)\hat k$$

So that

$$\vec a\cdot\vec b=\sqrt{p^2-4}\tan A+p\tan B+\sqrt{p^2+4}\tan C$$

Let θ be the angle between $\vec a$ and $\vec b$.

$$36p^2=(\vec a\cdot\vec b)^2$$

$$=|\vec a|^2|\vec b|^2\cos^2\theta\le|\vec a|^2|\vec b|^2$$

$$=(p^2-4+p^2+p^2+4)(\tan^2 A+\tan^2 B+\tan^2 C)$$

$$=3p^2(\tan^2 A+\tan^2 B+\tan^2 C)$$

Hence,

$$\tan^2 A+\tan^2 B+\tan^2 C\ge 12$$

and is equal to 12 when θ is either 0 or π.

118. (a,c) : Let θ be the angle between $\vec a$ and $\vec b$. Then

$$\vec a\cdot\vec b=|\vec a||\vec b|\cos\theta=\cos\theta$$

Now $|\vec\alpha|^2=|\vec a-(\vec a\cdot\vec b)\vec b|^2$

$$=|\vec a|^2+|\vec a\cdot\vec b)^2\vec b|^2-2(\vec a\cdot\vec b)^2$$

$$=1+\cos^2\theta-2\cos^2\theta=\sin^2\theta$$

$$=(|\vec a||\vec b|\sin\theta)^2\quad(\because\ |\vec a|=|\vec b|=1)$$

$$=|\vec a\times\vec b|^2=|\vec\beta|^2$$

Therefore

$$|\vec\beta|=|\vec\alpha|$$

Hence, (a) is correct. Now

$$\vec{\alpha}\cdot\vec{b}=[\vec{a}-(\vec{a}\cdot\vec{b})\vec{b}]\cdot\vec{b}$$

$$=\vec{a}\cdot\vec{b}-(\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{b})$$

$$=\vec{a}\cdot\vec{b}-\vec{a}\cdot\vec{b}\qquad(\because\ |\vec{b}|=1)$$

$$=0$$

Since

$$|\vec{\alpha}|+|\vec{\alpha}\cdot\vec{b}|=|\vec{\alpha}|+0=|\vec{\alpha}|=|\vec{\beta}|$$

Therefore (c) is correct.

119. (c) : Let E_1, E_2 and E_3 be the events of the critics giving favourable remarks. Then

$$P(E_1)=\frac{5}{7},P(E_2)=\frac{4}{7}\text{ and }P(E_3)=\frac{3}{7}$$

Let E be the event that majority reviewed favourably.

Therefore

$$E=(E_1\cap E_2\cap\bar{E}_3)\cup(\bar{E}_1\cap E_2\cap E_3)$$
$$\cup(E_1\cap\bar{E}_2\cap E_3)\cup(E_1\cap E_2\cap E_3)$$

Hence

$$P(E)=P(E_1)P(E_2)P(\bar{E}_3)+P(\bar{E}_1)P(E_2)P(E_3)$$
$$+P(E_1)P(\bar{E}_2)P(E_3+P(E_1)P(E_2)P(E_3)$$

$$=\left[\frac{5}{7}\times\frac{4}{7}\times\left(1-\frac{3}{7}\right)\right]+\left[\left(1-\frac{5}{7}\right)\times\frac{4}{7}\times\frac{3}{7}\right]+$$
$$\left[\frac{5}{7}\times\left(1-\frac{4}{7}\right)\times\frac{3}{7}\right]+\left[\frac{5}{7}\times\frac{4}{7}\times\frac{3}{7}\right]$$

$$=\left(\frac{5}{7}\times\frac{4}{7}\times\frac{4}{7}\right)+\left(\frac{2}{7}\times\frac{4}{7}\times\frac{3}{7}\right)+\left(\frac{5}{7}\times\frac{3}{7}\times\frac{3}{7}\right)+$$
$$\left(\frac{5}{7}\times\frac{4}{7}\times\frac{3}{7}\right)$$

$$=\frac{80+24+45+60}{7\times7\times7}=\frac{209}{343}$$

120. (d) : Let B_j be the number of black balls transferred (j = 0, 1, 2, 3). B is the event of drawing a black ball. Therefore

$$P(B_0)=\frac{{}^5C_4}{{}^8C_4}=\frac{5}{70}$$

$$P(B_1)=\frac{{}^3C_1\times{}^5C_3}{{}^8C_4}=\frac{30}{70}$$

$$P(B_2)=\frac{{}^3C_2\times{}^5C_2}{{}^8C_4}=\frac{30}{70}$$

$$P(B_3)=\frac{{}^3C_3\times{}^5C_1}{{}^8C_4}=\frac{5}{70}$$

Also

$P(B/B_0)=0$ ($\because$ no black ball is transferred)

$$P(B/B_1)=\frac{1}{4},\quad P(B/B_2)=\frac{2}{4}$$

$$P(B/B_3)=\frac{3}{4}$$

Therefore by Bayes' theorem,

$$P(B_3/B)=\frac{P(B_3)P(B/B_3)}{\sum_{i=0}^{3}P(B_i)P(B/B_i)}$$

$$=\frac{\frac{5}{70}\times\frac{3}{4}}{\frac{5}{70}\times0+\frac{30}{70}\times\frac{1}{4}+\frac{30}{70}\times\frac{2}{4}+\frac{5}{70}\times\frac{3}{4}}$$

$$=\frac{15}{30+60+15}=\frac{15}{105}=\frac{1}{7}$$

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