VITEEE
Practice Paper-2

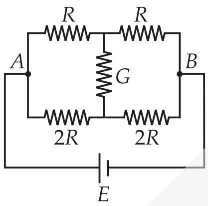
Exam Date
10th to 21st April

Time : 2 Hours 30 Minutes *Max. Marks : 120*

PHYSICS

1. A resistor has a colour code of green, blue, brown and silver. What is its resistance?
(a) $56\ \Omega \pm 5\%$ (b) $560\ \Omega \pm 10\%$
(c) $560\ \Omega \pm 5\%$ (d) $5600\ \Omega \pm 10\%$

2. Consider the following statements regarding the network shown in the figure



(1) The equivalent resistance of the network between points A and B is independent of value of G.
(2) The equivalent resistance of the network between points A and B is $\frac{4}{3}R$.
(3) The current through G is zero.
Which of the above statements is/are true?
(a) (1) only (b) (2) only
(c) (2) and (3) (d) (1), (2) and (3)

3. Particles that interact by the strong force are called
(a) leptons (b) hadrons
(c) muons (d) electrons

4. The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply. How much electrical energy is stored by the capacitor?
(a) 2.55×10^{-6} J (b) -2.55×10^{-6} J
(c) -2.05×10^{-6} J (d) 2.05×10^{-6} J

5. The flux entering and leaving a closed surface are 5×10^5 and 4×10^5 MKS units respectively, then the charge inside the surface will be
(a) -8.85×10^{-7} C (b) 8.85×10^{-7} C
(c) 8.85×10^{7} C (d) 6.85×10^{-7} C

6. Which of the following is not made by quarks?
(a) Neutron (b) Positron
(c) Proton (d) π-meson

7. Rising and setting sun appears to be reddish because of
(a) diffraction
(b) scattering due to dust particles and air molecules
(c) refraction
(d) polarization.

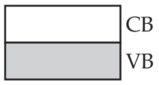
8. A conducting sphere of radius 10 cm is charged with 10 μC. Another uncharged sphere of radius 20 cm is allowed to touch it. After that if the spheres are separated, the surface density of charges on the spheres will be in the ratio of
(a) 1 : 4 (b) 1 : 2
(c) 1 : 3 (d) 2 : 1

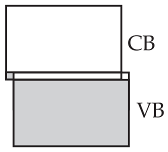
9. A 4 m long wire of resistance 8 Ω is connected in series with a battery of emf 2 V and a resistor of 7 Ω. The internal resistance of the battery is 1 Ω. What is the potential gradient along the wire?
(a) $1.00\ V\ m^{-1}$ (b) $0.75\ V\ m^{-1}$
(c) $0.50\ V\ m^{-1}$ (d) $0.25\ V\ m^{-1}$

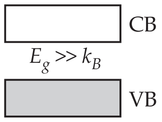
10. Excitation energy of a hydrogen like ion in its first excitation state is 40.8 eV. Energy needed to remove the electron from the ion in ground state is
(a) 54.4 eV (b) 13.6 eV
(c) 40.8 eV (d) 27.2 eV

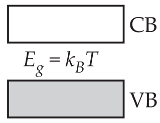
11. Two coils have a mutual inductance 0.005 H. The current changes in the first coil according to equation $I = I_0 \sin\omega t$, where $I_0 = 10$ A and $\omega = 100\pi$ rad s^{-1}. The maximum value of emf in the second coil is (in V)
(a) 2π (b) 5π
(c) π (d) 4π

12. Which of the energy band diagrams shown below corresponds to that of a semiconductor?

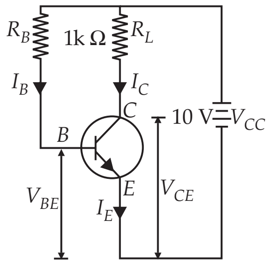
(a) 

(b) 

(c) 

(d) 

13. In the circuit shown in figure, the current gain, β = 100 for the transistor. What would be the base resistance R_B so that V_{CE} = 5V? (Neglect V_{BE}).

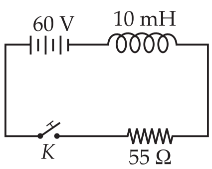


(a) 2×10^3 Ω (b) 2×10^5 Ω
(c) 1×10^6 Ω (d) 500 Ω

14. In the propagation of electromagnetic waves, the angle between the direction of propagation and plane of polarisation is
(a) 0° (b) 45°
(c) 90° (d) 180°

15. A long straight wire carrying a current of 30 A is placed in an external uniform magnetic field of induction 4×10^{-4} T. The magnetic field is acting parallel to the direction of current. The magnitude of the resultant magnetic induction in tesla at a point 2 cm away from the wire is
($\mu_0 = 4\pi \times 10^{-7}$ H m^{-1})
(a) 10^{-4} (b) 3×10^{-4}
(c) 5×10^{-4} (d) 6×10^{-4}

16. The rate of change of current is 500 A s^{-1} at the instant the key is pressed in the circuit shown in figure. The current through the circuit is



(a) 2 A (b) 1 A
(c) 0.5 A (d) 1.5 A

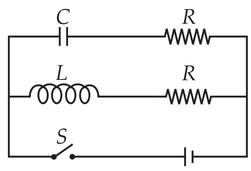
17. One of the devices to produce plane polarised light is
(a) nicol prism (b) mirror
(c) biprism (d) half wave plate

18. In an electromagnetic wave, the electric and magnetic fields are 100 V m^{-1} and 0.265 A m^{-1}. The maximum energy flow will be
(a) 79 W m^{-2} (b) 13.2 W m^{-2}
(c) 53.0 W m^{-2} (d) 26.5 W m^{-2}

19. There are three wavelengths : 10^{-8} m, 10^{-2} m, 10^{8} m. Their respective names are
(a) visible rays, γ-rays, ultraviolet rays
(b) ultraviolet, microwaves, radiowaves
(c) X-rays, visible rays, radiowaves
(d) radiowaves, X-rays, microwaves.

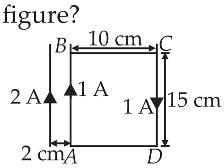
20. A Geiger counter is able to provide an indirect measure of radioactivity because radiation has a property of
(a) ionization
(b) making matter glow in the dark
(c) fogging photographic film
(d) altracting electrons

21. In the circuit shown in the figure, the switch S is closed at time $t = 0$.
$\left(\text{Given}, R = \sqrt{\frac{L}{C}}\right)$
The current through the capacitor and inductor will be equal at time t equals



(a) RC (b) $RC \ln 2$
(c) $\frac{1}{RC \ln 2}$ (d) LR

22. What is the net force on the rectangular coil shown in figure?



(a) 25×10^{-7} N towards wire
(b) 25×10^{-7} N away from wire
(c) 35×10^{-7} N towards wire
(d) 35×10^{-7} N away from wire

23. An electron having momentum 2.4×10^{-23} kg m s^{-1} enters a region of uniform magnetic field of 0.15 T. The field vector makes an angle of 30° with the initial velocity vector of the electron. The radius of the helical path of the electron in the field shall be

(a) 2 mm (b) 1 mm
(c) $\frac{\sqrt{3}}{2}$ mm (d) 0.5 mm

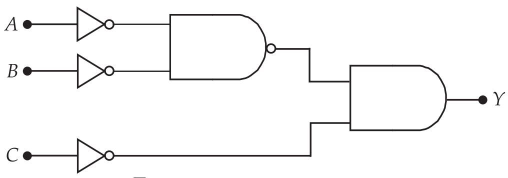
24. An electron is moving in a cyclotron at a speed of 3.2×10^7 m s^{-1} in a magnetic field of 5×10^{-4} T perpendicular to it. What is the frequency of this electron?
($e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg)
(a) 1.4×10^5 Hz (b) 1.4×10^7 Hz
(c) 1.4×10^6 Hz (d) 1.4×10^9 Hz

25. A ray of light moves from denser to rarer medium. Which of the following is correct?
(a) Energy increases
(b) Frequency increases
(c) Phase change by 90°
(d) Velocity increases

26. A 150 ohm resistor and an inductance L are connected in series to a 200 V, 50 Hz source of negligible impedance. The current comes to 1.0 A. When the source is changed to 400 V, 100 Hz, the current will be
(a) less than 1.0 A
(b) 1.0 A
(c) between 1 A and 2 A
(d) between 4 A and 2 A

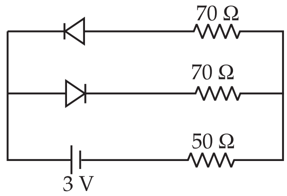
27. The activity of a radioactive sample is measured as N_0 counts per minute at $t = 0$ and N_0/e counts per minute at $t = 5$ minutes. The time (in minutes) at which the activity reduces to half its value is
(a) $\log_e \frac{2}{5}$ (b) $\frac{5}{\log_e 2}$
(c) $5\log_{10} 2$ (d) $5\log_e 2$

28. The output Y of the logic circuit as shown in figure is



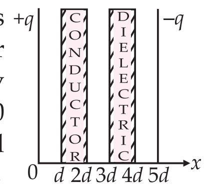
(a) $(A+B)\bar{C}$ (b) $(A+C)\bar{B}$
(c) $(B+C)\bar{A}$ (d) $A+B+C$

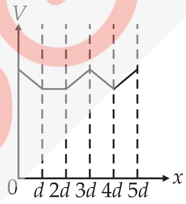
29. The circuit shown in the figure contains two diodes each with a forward resistance of 30 Ω and with infinite backward resistance. If the battery is 3 V, the current through the 50 Ω resistance (in ampere) is



(a) zero (b) 0.01
(c) 0.02 (d) 0.03

30. The distance between plates of a parallel plate capacitor is $5d$. The positively charged plate is at $x = 0$ and negatively charged plate is at $x = 5d$. Two slabs one of conductor and the other of a dielectric of same thickness d are inserted between the plates as shown in figure. Potential (V) versus distance (x) graph will be



(a)


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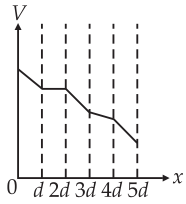

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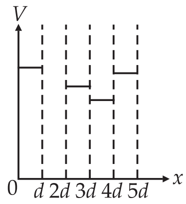
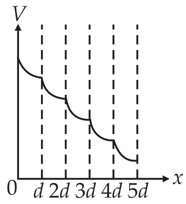
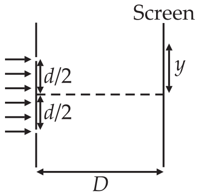


(d)


31. Let K_1 be the maximum kinetic energy of photoelectrons emitted by light of wavelength λ_1 and K_2 corresponding to wavelength λ_2. If $\lambda_1 = 2\lambda_2$ then
(a) $2K_1 = K_2$ (b) $K_1 = 2K_2$
(c) $K_1 < K_2/2$ (d) $K_1 > 2K_2$

32. A rigid circular loop of radius r and mass m lies in the x-y plane on a flat table and has a current I flowing in it. At this particular place, the earth's magnetic field is $\vec{B} = B_x\hat{i} + B_z\hat{k}$. What is the value I so that one edge of the loop lifts from the table?
(a) $\frac{mg}{\pi r\sqrt{B_x^2 + B_z^2}}$ (b) $\frac{mg}{\pi r B_z}$
(c) $\frac{mg}{\pi r B_x}$ (d) $\frac{mg}{\pi r\sqrt{B_x B_z}}$

33. In the Young's double slit experiment apparatus shown in figure, the ratio of maximum to minimum intensity on the screen is 9. The wavelength of light used is λ, then the value of y is



(a) $\frac{\lambda D}{d}$ (b) $\frac{\lambda D}{2d}$

(c) $\frac{\lambda D}{3d}$ (d) $\frac{\lambda D}{4d}$

34. Find the phase velocity of electromangetic wave having electron density and frequency for D layer, N = 400 electron per c.c., υ = 200 kHz.

(a) 3×10^8 m s^{-1} (b) 3.4×10^8 m s^{-1}

(c) 6.9×10^8 m s^{-1} (d) 1.1×10^9 m s^{-1}

35. The magnifying power of a telescope is 9. When it is adjusted for parallel rays, the distance between the objective and the eye-piece is found to be 20 cm. The focal lengths of the lenses are

(a) 18 cm, 2 cm (b) 11 cm, 9 cm

(c) 10 cm, 10 cm (d) 15 cm, 5 cm

36. An object is placed 30 cm to the left of a diverging lens whose focal length is of magnitude 20 cm. Which one of the following correctly states the nature and position of the virtual image formed?

	Nature of image	Distance from lens
(a)	inverted, enlarged	60 cm to the right
(b)	erect, diminished	12 cm to the left
(c)	inverted, enlarged	60 cm to the left
(d)	erect, diminished	12 cm to the right

37. The angular momentum of electron in 3d orbital of an atom is

(a) $\sqrt{2}\left(\frac{h}{2\pi}\right)$ (b) $\sqrt{3}\left(\frac{h}{2\pi}\right)$

(c) $\sqrt{6}\left(\frac{h}{2\pi}\right)$ (d) $\sqrt{12}\left(\frac{h}{2\pi}\right)$

38. The binding energy of an electron in the ground state of He is equal to 24.6 eV. The energy required to remove both the electrons is

(a) 49.2 eV (b) 24.6 eV

(c) 38.2 eV (d) 79.0 eV

39. A convex lens of focal length 0.15 m is made of a material of refractive index 3/2. When it is placed in a liquid, its focal length is increased by 0.225 m. The refractive index of the liquid is

(a) $\frac{7}{4}$ (b) $\frac{5}{4}$

(c) $\frac{9}{4}$ (d) $\frac{3}{2}$

40. A transparent thin plate of a polaroid is placed on another similar plate such that the angle between their axes is 30°. The intensities of the emergent and the unpolarized incident light will be in the ratio of

(a) 1 : 4 (b) 1 : 3

(c) 3 : 4 (d) 3 : 8

CHEMISTRY

41. In cyclotrimetaphosphoric acid, number of P—O—P bonds, P = O bonds and P—OH bonds are respectively

(a) 6, 3, 3 (b) 5, 0, 3

(c) 4, 3, 0 (d) 3, 3, 3

42. Which of the following carbonyl compounds is most polar?

(a) $C_2H_5—C(=O)—C_2H_5$ (b) $CH_3—C(=O)—CH_3$

(c) $CH_3—C(=O)—H$ (d) $H—C(=O)—H$

43. In an irreversible process taking place at constant T and P and in which only pressure-volume work is being done, the change in Gibbs free energy (dG) and change in entropy (dS), satisfy the criteria

(a) $(dS)_{V,E} < 0, (dG)_{T,P} < 0$

(b) $(dS)_{V,E} > 0, (dG)_{T,P} < 0$

(c) $(dS)_{V,E} = 0, (dG)_{T,P} = 0$

(d) $(dS)_{V,E} = 0, (dG)_{T,P} > 0$

44. The reagent with which both acetaldehyde and acetone react easily is

(a) Fehling's reagent (b) Grignard reagent

(c) Schiff's reagent (d) Tollens' reagent.

45. If the solubility of $PbCl_2$ at 25°C is 6.3×10^{-3} mol L^{-1} its solubility product at that temperature is

(a) $(6.3 \times 10^{-3}) \times (6.3 \times 10^{-3})^2$

(b) $(6.3 \times 10^{-3}) \times (12.6 \times 10^{-3})^2$

(c) $(6.3 \times 10^{-3}) \times (12.6 \times 10^{-3})$

(d) $(12.6 \times 10^{-3}) \times (12.6 \times 10^{-3})$

46. Which of the following organic compounds gives positive Fehling's test as well as iodoform test?
(a) Methanal (b) Ethanol
(c) Propanone (d) Ethanal

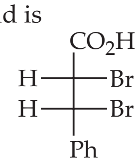
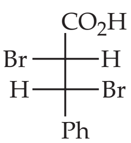
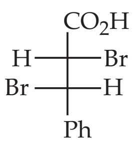
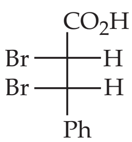
47. What is the maximum number of emission lines when the excited electron of a hydrogen atom in $n = 6$ drops to ground state?
(a) 6 (b) 15
(c) 30 (d) 10

48. Which one of the following species is not a pseudohalide?
(a) CNO^- (b) $RCOO^-$
(c) OCN^- (d) NNN^-

49. The number of radial nodes of 3s and 2p orbitals are respectively
(a) 2, 0 (b) 0, 2
(c) 1, 2 (d) 2, 1

50. The number of structural and configurational isomers of a bromo compound C_5H_9Br, formed by the addition of HBr to pent-2-yne respectively are
(a) 1 and 2 (b) 2 and 4
(c) 4 and 2 (d) 2 and 1

51. Alkali halides do not show Frenkel defect because
(a) cations and anions have almost equal size
(b) there is a large difference in size of cations and anions
(c) cations and anions have low coordination number
(d) anions cannot be accommodated in voids.

52. The structure of 2R, 3S-dibromocinnamic acid is
(a) 
(b) 
(c) 
(d) 

53. The EMF of the cell,
$Zn \mid Zn^{2+}$ (0.01 M) $\mid\mid Fe^{2+}$ (0.001M) $\mid Fe$
at 298 K is 0.2905 V then the value of equilibrium constant for the cell reaction is
(a) $e^{\frac{0.32}{0.0295}}$ (b) $10^{\frac{0.32}{0.0295}}$
(c) $10^{\frac{0.26}{0.0295}}$ (d) $10^{\frac{0.32}{0.0591}}$

54. Interhalogen compounds are more reactive than the individual halogens because
(a) they are prepared by direct combination of halogens
(b) $X - X'$ bond is weaker than $X - X$ or $X' - X'$ bond
(c) they are thermally more stable than halogens
(d) there is a large difference in their electronegativities.

55. Ground state energy of H-atom is $(-E_1)$, the velocity of photoelectrons emitted when photon of energy E_2 strikes stationary Li^{2+} ion in ground state will be
(a) $v = \sqrt{\frac{2(E_2 - E_1)}{m}}$ (b) $v = \sqrt{\frac{2(E_2 + 9E_1)}{m}}$
(c) $v = \sqrt{\frac{2(E_2 - 9E_1)}{m}}$ (d) $v = \sqrt{\frac{2(E_2 - 3E_1)}{m}}$

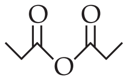
56. Oils are converted into fats by
(a) hydration (b) decarboxylation
(c) hydrogenation (d) dehydrogenation.

57. Glycol is added to aviation petrol because
(a) it prevents freezing of petrol
(b) it minimises the loss of petrol
(c) it increases the efficiency of fuel
(d) it prevents the engine from heating up.

58. Which of the following expressions correctly represents the equivalent conductance at infinite dilution of $Al_2(SO_4)_3$? Given that $\overset{\circ}{\Lambda}_{Al^{3+}}$ and $\overset{\circ}{\Lambda}_{SO_4^{2-}}$ are the equivalent conductances at infinite dilution of the respective ions.
(a) $2\overset{\circ}{\Lambda}_{Al^{3+}} + 3\overset{\circ}{\Lambda}_{SO_4^{2-}}$ (b) $\overset{\circ}{\Lambda}_{Al^{3+}} + \overset{\circ}{\Lambda}_{SO_4^{2-}}$
(c) $(\overset{\circ}{\Lambda}_{Al^{3+}} + \overset{\circ}{\Lambda}_{SO_4^{2-}}) \times 6$ (d) $\frac{1}{3}\overset{\circ}{\Lambda}_{Al^{3+}} + \frac{1}{2}\overset{\circ}{\Lambda}_{SO_4^{2-}}$

59. The correct order of ionic radii of Ce, La, Pm and Yb in +3 oxidation state is
(a) $La^{3+} < Pm^{3+} < Ce^{3+} < Yb^{3+}$
(b) $Yb^{3+} < Pm^{3+} < Ce^{3+} < La^{3+}$
(c) $La^{3+} < Ce^{3+} < Pm^{3+} < Yb^{3+}$
(d) $Yb^{3+} < Ce^{3+} < Pm^{3+} < La^{3+}$

60. The IUPAC name of the following compound

 is

(a) propionic anhydride
(b) dipropanoic anhydride
(c) ethoxy propanoic acid
(d) propanoic anhydride.

61. Consider the reactions,

$NO_2 \rightleftharpoons \frac{1}{2}N_2 + O_2,\ K_1$

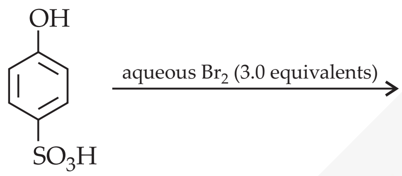
$N_2O_4 \rightleftharpoons 2NO_2,\quad K_2$

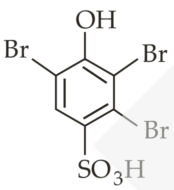
Give the equilibrium constant for the formation of N_2O_4 from N_2 and O_2.

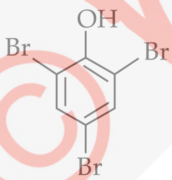
(a) $\frac{1}{K_1^2}+\frac{1}{K_2}$ (b) $\frac{1}{2K_1}+\frac{1}{K_2}$

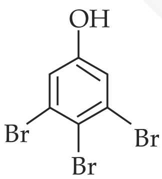
(c) $\sqrt{\frac{1}{K_1K_2}}$ (d) $\frac{K_2}{K_1}$

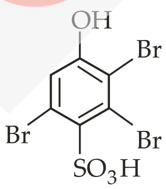
62. The major product of the following reaction is



(a) 

(b) 

(c) 

(d) 

63. Which of the following arrangements shows schematic alignment of magnetic moments of antiferromagnetic substances?

(a) ↑ ↑ ↑ ↑ ↑ ↑

(b) ↓ ↓ ↓ ↓ ↓ ↓

(c) ↑ ↑ ↓ ↑ ↑ ↓

(d) ↑ ↓ ↑ ↓ ↑ ↓

64. The ease of liquefaction of noble gases decreases in the order

(a) He > Ne > Ar > Kr > Xe
(b) Xe > Kr > Ar > Ne > He
(c) Kr > Xe > He > Ar > Ne
(d) Ar > Kr > Xe > He > Ne

65. Consider the following sets of quantum numbers:

	n	l	m	s
(i)	3	0	0	+1/2
(ii)	2	2	1	+1/2
(iii)	4	3	–2	–1/2
(iv)	1	0	–1	–1/2
(v)	3	2	3	+1/2

Which of the following sets of quantum numbers is not possible?

(a) (i), (ii), (iii) and (iv)
(b) (ii), (iv) and (v)
(c) (i) and (iii)
(d) (ii), (iii) and (iv)

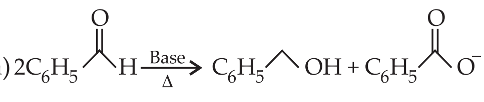
66. Which of the following is not formed when glycerol reacts with HI?

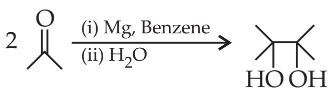
(a) $CH_2{=}CH-CH_2I$
(b) $CH_2OH-CHI-CH_2OH$
(c) $CH_3-CH{=}CH_2$
(d) $CH_3-CHI-CH_3$

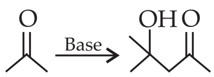
67. For preparing a buffer solution of pH = 6 by mixing sodium acetate and acetic acid, the ratio of the concentration of salt and acid should be ($K_a = 10^{-5}$)

(a) 1 : 10 (b) 10 : 1
(c) 100 : 1 (d) 1 : 100

68. Which one of the following is hydride transfer reaction?

(a) 

(b) 

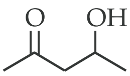
(c) 

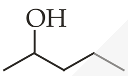
(d) 

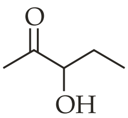
69. Which of the following compounds can be detected by Molisch's test?
(a) Sugars
(b) Amines
(c) Primary alcohols
(d) Nitro compounds

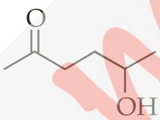
70. Hess's law is applicable for the determination of heat of
(a) transition (b) formation
(c) reaction (d) all of these.

71. Which of the following statements is true regarding main cause of lanthanide contraction?
(a) Poor shielding of 4*f*-electron by another in the subshell
(b) Poor shielding of 5*d*-electrons
(c) Effective shielding of 4*f*-electrons
(d) Effective shielding of 5*d*-electrons by 4*f*-electrons

72. Which of the following will be most readily dehydrated in acidic conditions?
(a) 

(b) 

(c) 

(d) 

73. An organic compound with the formula $C_6H_{12}O_6$ forms a yellow crystalline solid with phenylhydrazine and gives a mixture of sorbitol and mannitol when reduced with sodium. Which among the following could be the compound?
(a) Fructose (b) Glucose
(c) Mannose (d) Sucrose

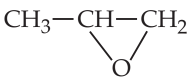
74. Which of the following statements is incorrect?
(a) $t_{1/2} \propto a$, for zero order reaction.
(b) $t_{1/2}$ is independent of a, for first order reaction.
(c) $t_{1/2} \propto \frac{1}{a^2}$, for third order reaction.
(d) $t_{1/2} \propto \frac{1}{a^{1-n}}$, for n^{th} order reaction

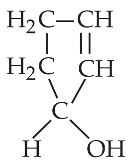
75. An ester (*A*) with molecular formula $C_9H_{10}O_2$ was treated with excess of CH_3MgBr and the compound so formed was treated with conc. H_2SO_4 to form olefin (*B*). Ozonolysis of (*B*) gave ketone with formula C_8H_8O which shows positive iodoform test. The structure of (*A*) is
(a) $C_6H_5COOC_2H_5$
(b) $CH_3OCH_2COC_6H_5$
(c) $CH_3COC_6H_4COCH_3$
(d) $C_6H_5COOC_6H_5$

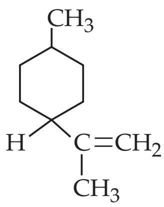
76. In the reduction of nitrobenzene, which of the following is the intermediate?
(a) $C_6H_5-N=O$
(b) $C_6H_5NH-NHC_6H_5$
(c) $C_6H_5-N=N-C_6H_5$
(d) $C_6H_5-N=\overset{\overset{O}{\uparrow}}{N}-C_6H_5$

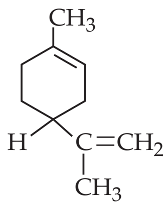
77. Which of the following ligands will not show chelation?
(a) EDTA
(b) DMG
(c) Ethene-1, 2-diamine
(d) SCN^-

78. pK_a of a weak acid is 5.76 and pK_b of a weak base is 5.25. What will be the pH of the salt formed by the two?
(a) 7.255 (b) 7.005
(c) 10.225 (d) 4.255

79. Which of the following compounds does not contain chiral carbon?
(a) 

(b) 

(c) 

(d) 

80. Increasing order of reactivity of the following alkyl halides in the Williamson's synthesis is
I. $CH_2=CHCH_2Cl$ II. $CH_3CH_2CH_2Br$
III. $(CH_3)_3CCH_2Br$ IV. $CH_3CH_2CH_2Cl$
(a) II < III < IV < I (b) III < II < IV < I
(c) IV < III < I < II (d) III < IV < I < II

MATHEMATICS

81. If the point of intersection of the lines $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$ and $\frac{x-2}{2}=\frac{y-6}{2}=\frac{z-3}{4}$ is (x, y, z), then $y + x$ is.
(a) 9 (b) –8
(c) 8 (d) 1

82. If $|\vec{a}|=\sqrt{6}$ and $|\vec{b}|=\sqrt{5}$, then $[(\vec{a}\times\vec{b})\times\vec{b}]\times\vec{b}$ is equal to
(a) $5(\vec{b}\times\vec{a})$ (b) $5(\vec{a}\times\vec{b})$
(c) $6(\vec{a}\times\vec{b})$ (d) $6(\vec{b}\times\vec{a})$

83. A bag contains 4 balls of unknown colours. A ball is drawn at random from it and is found to be white. The probability that all the balls in the bag are white is
(a) $\frac{4}{5}$ (b) $\frac{1}{5}$
(c) $\frac{3}{5}$ (d) $\frac{2}{5}$

84. A natural number is selected at random from the first 100 natural numbers. Let A, B and C denote the events of selection of even number, a multiple of 3 and a multiple of 5, respectively. Then
(a) $P(A\cap B)=\frac{4}{25}$ (b) $P(B\cap C)=\frac{3}{50}$
(c) $P(C\cap A)=\frac{1}{10}$ (d) All of these

85. Find the distance between the following parallel planes
$2x - y + 2z + 3 = 0$ and $4x - 2y + 4z + 5 = 0$
(a) 2/3 (b) 4
(c) 1/6 (d) 2/5

86. Find the angle between the pair of lines given by
$$\vec{r}=3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$$
and $\vec{r}=5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$
(a) $\cos^{-1}\left(\frac{19}{21}\right)$ (b) $\cos^{-1}\left(\frac{21}{19}\right)$
(c) $\cos\left(\frac{19}{20}\right)$ (d) $\cos^{-1}\left(\frac{20}{21}\right)$

87. Solve the following differential equation
$\frac{dy}{dx}+\frac{1+\cos 2y}{1-\cos 2x}=0.$
(a) $\tan x = \cot y + C$ (b) $\cot x = \tan y + C$
(c) $\cot y = \tan x + C$ (d) $\tan y = \cot x + C$

88. Solve the following differential equation
$(1 + x^2)dy + 2xy\,dx = \cot x\,dx$, $(x \neq 0)$.
(a) $\sin x + C$ (b) $\frac{\log|\sin x|}{1+x^2}+\frac{C}{1+x^2}$
(c) $1 + x^2 + C$ (d) $\log|x| + C$

89. Find the area bounded by curves $\{(x, y) : y \geq x^2 \text{ and } y \leq |x|\}$.
(a) 3 sq. units (b) 5 sq. units
(c) 2 sq. units (d) 1/3 sq. units

90. Find the area bounded by the curves $y = \sin x$ between $x = 0$ and $x = 2\pi$.
(a) 4 sq. units (b) 6 sq. units
(c) 2 sq. units (d) 8 sq. units

91. If $f(x)=\cos^{-1}\left(\frac{x^{-1}-x}{x^{-1}+x}\right)$, then $f'(x)$ is
(a) odd (b) even
(c) periodic (d) None of these

92. If $y=\cot^{-1}\left(\frac{\ln(e/x^2)}{\ln(ex^2)}\right)+\cot^{-1}\left(\frac{\ln(ex^4)}{\ln(e^2/x^2)}\right)$, then $\frac{d^2y}{dx^2}$ equals
(a) –1 (b) 0
(c) 1 (d) 2

93. If $f_n(x)=e^{f_{n-1}(x)}\ \forall\, n\in\mathbf{N}$ and $f_0(x)=x$, then $f_n'(x)$ equals
(a) $\sum_{r=1}^{n} f_r(x)$ (b) $\prod_{r=1}^{n} f_r(x)$
(c) $\prod_{r=1}^{n-1} f_r'(x)$ (d) None of these

94. The function $f(x)=\sin\left(\frac{\pi}{x}\right)$ is strictly decreasing in the interval
(a) $(2n + 3, 2n + 5)$, $n \in I$
(b) $\left(\frac{3}{6n+2},\frac{3}{6n+1}\right)$, $n\in I$
(c) $\left(\frac{2}{4n+1},\frac{2}{4n-1}\right)$, $n\in I$
(d) None of these

95. For $x > 1$, $y = \ln x$ satisfies the inequality
(a) $y < x - 1$ (b) $y<1-\frac{1}{x}$
(c) $y < x^2 - 1$ (d) None of these

96. For any function $f(x)$, if $f'(a)=f''(a)=\ldots\ldots=f^{(n-1)}(a)=0$ but $f^{(n)}(a)\neq 0$, then $f(x)$ has a minima at $x = a$ if
(a) n is even and $f^{(n)}(a) > 0$
(b) n is even and $f^{(n)}(a) < 0$
(c) n is odd and $f^{(n)}(a) > 0$
(d) n is odd and $f^{(n)}(a) < 0$

97. The largest term in the sequence $a_n = \frac{n^2}{n^3+200}$, is
(a) a_1 (b) a_7
(c) a_8 (d) None of these

98. The value of $\int_0^{\pi}[\tan x]dx$ ([·] denotes integral part), is equal to
(a) $-\pi/2$ (b) 0
(c) –1 (d) None of these

99. If ω is an imaginary cube root of unity, then the value of $\frac{1}{1+2\omega}+\frac{1}{2+\omega}-\frac{1}{1+\omega}$ is
(a) –2 (b) –1
(c) 1 (d) 0

100. If $(2+i)(2+2i)(2+3i)\ldots\ldots(2+ni) = x+iy$, then $5.\,8.\,13.\ldots\ldots(4+n^2)$ is equal to
(a) x^2-y^2 (b) x^2+y^2
(c) x^4-y^4 (d) x^4+y^4

101. The value of the expression
$$\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+\left(3+\frac{1}{\omega}\right)\left(3+\frac{1}{\omega^2}\right)+\ldots\ldots+\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right),$$
where ω is an imaginary cube root of unity is
(a) $\frac{n(n^2+3)}{3}$ (b) $\frac{n(n^2+2)}{3}$
(c) $\frac{n(n^2+1)}{3}$ (d) None of these

102. If a straight line through the point $P(3, 4)$ makes an angle $\frac{\pi}{6}$ with x-axis and meets the line $12x+5y+10=0$ at Q, find the length of PQ.
(a) 132 (b) $12\sqrt{3}+5$
(c) $\frac{-132}{12\sqrt{3}+5}$ (d) 5

103. Find the orthocentre of the triangle whose sides have equations $x-2=0$, $y-5=0$ and $5x+2y-10=0$.
(a) (5, 2) (b) (0, 2)
(c) (5, 0) (d) (2, 5)

104. The lengths of the tangents from any point on the circle $15x^2+15y^2-48x+64y=0$ to the two circles $5x^2+5y^2-24x+32y+75=0$ and $5x^2+5y^2-48x+64y+300=0$ are in the ratio
(a) 1 : 2 (b) 2 : 3
(c) 3 : 4 (d) None of these

105. If the point $P(4, -2)$ is the one end of the focal chord PQ of the parabola $y^2 = x$, then the slope of the tangent at Q is
(a) $-\frac{1}{4}$ (b) $\frac{1}{4}$
(c) 4 (d) – 4

106. The focus of the parabola $y^2-x-2y+2=0$ is
(a) $\left(\frac{1}{4},0\right)$ (b) (1, 2)
(c) $\left(\frac{3}{4},1\right)$ (d) $\left(\frac{5}{4},1\right)$

107. If P is a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with AA' as the major axis. Find the maximum value of the area of the triangle APA'.
(a) b (b) a^2
(c) ab (d) a

108. If e and e' be the eccentricities of a hyperbola and its conjugate, then $\frac{1}{e^2}+\frac{1}{e'^2}$ is equal to
(a) 0 (b) 1
(c) 2 (d) None of these

109. The number of solutions of the system of equations $3x-2y+z=5$, $6x-4y+2z=10$ and $9x-6y+3z=15$ is
(a) 0 (b) 1
(c) 2 (d) infinite.

110. If A is a square matrix satisfying the equation $A^2-4A-5I=0$, then $A^{-1}=$
(a) $A-4I$ (b) $\frac{1}{3}(A-4I)$
(c) $\frac{1}{4}(A-4I)$ (d) $\frac{1}{5}(A-4I)$

111. If p : It rains today, q : I go to the school, r : I shall meet my friends and s : I shall go for a movie, then which of the following is the proposition :
If does not rain or if I do not go to school, then I shall meet my friend and go for a movie.
(a) $\sim(p\wedge q)\Rightarrow(r\wedge s)$
(b) $(\sim p\wedge\sim q)\Rightarrow(r\wedge s)$
(c) $\sim(p\vee q)\Rightarrow(r\vee s)$
(d) None of these

112. If p : Ajay is tall
q : Ajay is intelligent
then the symbolic statement $\sim p\vee q$ means
(a) Ajay is not tall or he is intelligent.
(b) Ajay is tall or he is intelligent.
(c) Ajay is not tall and he is intelligent.
(d) Ajay is not tall then he is intelligent.

113. $(\sim(\sim p)) \wedge q$ is equal to

(a) $\sim p \wedge q$ (b) $p \wedge q$

(c) $\sim p \wedge \sim q$ (d) $p \wedge \sim q$

114. Negation of the compound proposition
If the Examination is difficult, then I shall pass if I study hard.

(a) The Examination is difficult and I study hard but I shall not pass.

(b) The Examination is not difficult and I study hard and I shall pass.

(c) The Examination is difficult and I study hard and I shall pass.

(d) None of these.

115. If $x^2 + y^2 = 1$, then

(a) $yy'' - (2y')^2 + 1 = 0$

(b) $yy'' + (y')^2 + 1 = 0$

(c) $yy'' - (y')^2 - 1 = 0$

(d) $yy'' + 2(y')^2 + 1 = 0$

116. The differential equation of all parabolas whose axes are parallel to y-axis, is

(a) $\frac{d^3y}{dx^3} = 0$ (b) $\frac{d^2y}{dx^2} = 0$

(c) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (d) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

117. The integrating factor of the differential equation $(y \log y)\, dx = (\log y - x)\, dy$ is

(a) $\frac{1}{\log y}$ (b) $\log(\log y)$

(c) $1 + \log y$ (d) $\log y$

118. If the vectors $\vec{a} = (2, \log_3 x, a)$ and $\vec{b} = (-3, a \log_3 x, \log_3 x)$ are inclined at an acute angle, then

(a) $a = 0$ (b) $a < 0$

(c) $a > 0$ (d) None of these

119. If $\vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors and

$$\Delta = \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix}, \text{ then}$$

(a) $\Delta = 0$

(b) $\Delta = 1$

(c) Δ = any non-zero value

(d) None of these

120. The p.c of the centre of the sphere $|\vec{r}|^2 + \vec{r}\cdot(\hat{i} + \hat{j} - \hat{k}) - 9 = 0$ is

(a) $\hat{i} + \hat{j} - \hat{k}$ (b) $\frac{1}{2}(\hat{i} + \hat{j} - \hat{k})$

(c) $-\frac{1}{2}(\hat{i} + \hat{j} - \hat{k})$ (d) $-\hat{i} + \hat{j} + \hat{k}$

ANSWER KEY

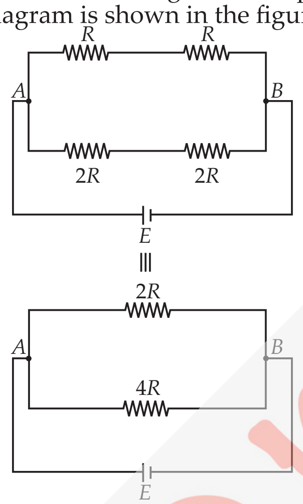
1.	(b)	**2.**	(d)	**3.**	(b)	**4.**	(a)	**5.**	(a)	**6.**	(b)	**7.**	(b)	**8.**	(d)
9.	(d)	**10.**	(a)	**11.**	(b)	**12.**	(d)	**13.**	(b)	**14.**	(a)	**15.**	(c)	**16.**	(b)
17.	(a)	**18.**	(d)	**19.**	(b)	**20.**	(a)	**21.**	(b)	**22.**	(a)	**23.**	(d)	**24.**	(b)
25.	(d)	**26.**	(c)	**27.**	(d)	**28.**	(a)	**29.**	(c)	**30.**	(b)	**31.**	(c)	**32.**	(c)
33.	(c)	**34.**	(c)	**35.**	(a)	**36.**	(b)	**37.**	(c)	**38.**	(d)	**39.**	(b)	**40.**	(d)
41.	(d)	**42.**	(d)	**43.**	(b)	**44.**	(b)	**45.**	(b)	**46.**	(d)	**47.**	(b)	**48.**	(b)
49.	(a)	**50.**	(b)	**51.**	(a)	**52.**	(d)	**53.**	(b)	**54.**	(b)	**55.**	(c)	**56.**	(c)
57.	(a)	**58.**	(b)	**59.**	(b)	**60.**	(d)	**61.**	(a)	**62.**	(b)	**63.**	(d)	**64.**	(b)
65.	(b)	**66.**	(b)	**67.**	(b)	**68.**	(a)	**69.**	(a)	**70.**	(d)	**71.**	(a)	**72.**	(a)
73.	(a)	**74.**	(d)	**75.**	(a)	**76.**	(a)	**77.**	(d)	**78.**	(a)	**79.**	(c)	**80.**	(d)
81.	(c)	**82.**	(a)	**83.**	(d)	**84.**	(d)	**85.**	(c)	**86.**	(a)	**87.**	(d)	**88.**	(b)
89.	(d)	**90.**	(a)	**91.**	(a)	**92.**	(b)	**93.**	(b)	**94.**	(c)	**95.**	(a,c)	**96.**	(a)
97.	(b)	**98.**	(a)	**99.**	(d)	**100.**	(b)	**101.**	(b)	**102.**	(c)	**103.**	(d)	**104.**	(a)
105.	(c)	**106.**	(d)	**107.**	(c)	**108.**	(b)	**109.**	(d)	**110.**	(d)	**111.**	(a)	**112.**	(a)
113.	(b)	**114.**	(a)	**115.**	(b)	**116.**	(a)	**117.**	(d)	**118.**	(d)	**119.**	(c)	**120.**	(c)

explanations

PHYSICS

1. **(b) :** Numbers corresponding to green, blue, brown and silver are 5, 6, 1 and 10% respectively. Therefore, the resistance of given resistor
$= 56 \times 10^1\ \Omega \pm 10\% = 560\ \Omega \pm 10\%$

2. **(d) :** As $\frac{R}{R} = \frac{2R}{2R}$, hence the given network is a balanced Wheatstone bridge. Therefore, no current flows through G, so equilvalent circuit diagram is shown in the figure.



Hence, the equivalent resistance of the network between points A and B is

$$R_{eq} = \frac{(2R)(4R)}{2R+4R} = \frac{4}{3}R$$

3. **(b)**

4. **(a) :** Here, $A = 90\ \text{cm}^2 = 90 \times 10^{-4}\ \text{m}^2$
$d = 2.5\ \text{mm} = 2.5 \times 10^{-3}\ \text{m}$,
$V = 400\ \text{V}$
Electrical energy stored $= \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$

$$= \frac{1}{2} \times \frac{8.85 \times 10^{-12} \times 90 \times 10^{-4}}{2.5 \times 10^{-3}}(400)^2$$
$= 2.55 \times 10^{-6}\ \text{J}$

5. **(a) :** Net flux leaving the surface,
$\Delta\phi = 4 \times 10^5 - 5 \times 10^5 = -10^5$ MKS
∴ Charge must be negative
$q = \phi\varepsilon_0 = -10^5 \times 8.85 \times 10^{-12}$
$= -8.85 \times 10^{-7}\ \text{C}$

6. **(b) :** Baryons are made of three quarks and mesons are made of one quark and one antiquark. On the other hand, leptons are not made of quarks.
Neutron and proton are baryons.
Positron is a lepton.
π-meson is a meson.

7. **(b)**

8. **(d) :** Common potential,

$$V = \frac{q_1+q_2}{C_1+C_2} = \frac{10+0}{4\pi\varepsilon_0(0.1+0.2)} = \frac{10}{4\pi\varepsilon_0(0.3)}$$

Charges after contact

$$q'_1 = C_1V = 4\pi\varepsilon_0(0.1) \times \frac{10}{4\pi\varepsilon_0(0.3)} = \frac{10}{3}\ \mu C$$

and $q'_2 = C_2V = 4\pi\varepsilon_0(0.2) \times \frac{10}{4\pi\varepsilon_0(0.3)} = \frac{20}{3}\ \mu C$

∴ The ratio of surface density of charges

$$\frac{\sigma_1}{\sigma_2} = \frac{q'_1}{4\pi r_1^2} \times \frac{4\pi r_2^2}{q'_2} = \frac{q'_1}{q'_2}\left(\frac{r_2}{r_1}\right)^2$$

$$= \frac{10}{3} \times \frac{3}{20}\left(\frac{0.2}{0.1}\right)^2 = \frac{1}{2} \times 4 = 2:1$$

9. **(d) :** Current in the potentiometer,

$$I = \frac{2}{8+7+1} = \frac{1}{8}\ \text{A}$$

Voltage drop across potentiometer wire

$$V = \frac{1}{8} \times 8 = 1\ \text{V}$$

∴ Potential gradient of potentiometer wire

$$\frac{V}{l} = \frac{1}{4} = 0.25\ \text{V m}^{-1}$$

10. **(a) :** Excitation energy

$$\Delta E = E_2 - E_1 = 13.6Z^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$40.8 = 13.6Z^2 \times \frac{3}{4}$$

or $Z = 2$
So, energy required to remove the electron from ground state

$$= +\frac{13.6Z^2}{(1)^2} = 13.6(2)^2 = 54.4\ \text{eV}$$

11. **(b) :** Induced emf in second coil,

$$\varepsilon_2 = M\frac{dI_1}{dt} = 0.005 \times \frac{d}{dt}(I_0 \sin\omega t)$$

$= 0.005 \times I_0\omega\cos\omega t$

∴ $\varepsilon_{2max} = 0.005 \times 10 \times 100\pi = 5\pi\ \text{V}$

12. (d) : In a semiconductor, the energy gap (E_g) between valence band and conduction band corresponds to thermal energy (k_BT) at room temperature (T).
Hence option (d) represent the correct diagram.

13. (b) : Here, $\beta = 100$, $V_{CE} = 5$ V, $V_{CC} = 10$ V

As $\beta = \frac{I_C}{I_B}$ or $I_B = \frac{I_C}{\beta} = \frac{I_C}{100}$ ...(i)

Also, $V_{CE} = V_{CC} - I_C R_L$

or $5\text{V} = 10\text{ V} - I_C \times 1000$

$\therefore\ I_C = \frac{5\text{V}}{1000\Omega} = 5 \times 10^{-3}$ A

and $I_B = \frac{5\times10^{-3}\text{A}}{100} = 5 \times 10^{-5}$ A (Using(i))

Thus, $R_B = \frac{V_{CC} - V_{BE}}{I_B}$

$= \frac{10\text{V}}{5\times10^{-5}\text{A}} = 2 \times 10^5\ \Omega$ (neglecting V_{BE})

14. (a) : Plane of vibration is perpendicular to direction of propagation and also perpendicular to plane of polarisation. Therefore, angle between plane of polarisation and direction of propagation is 0°.

15. (c) : $B_1 = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7})\times30}{2\pi\times0.02} = 3 \times 10^{-4}$

The direction of B_1 will be perpendicular to B_2 ($= 4 \times 10^{-4}$ T)

Hence, resultant magnetic field at given point is

$B = \sqrt{B_1^2 + B_2^2}$

$= [(3 \times 10^{-4})^2 + (4 \times 10^{-4})^2]^{1/2} = 5 \times 10^{-4}$ T.

16. (b) : Induced e.m.f. across L is

$\varepsilon = L\frac{dI}{dt} = 10^{-2} \times 500 = 5$ V

$\therefore$ Potential difference across R is,

$V = 60 - 5 = 55$ V

$\therefore\ I = \frac{V}{R} = \frac{55}{55} = 1$ A

17. (a) : A nicol prism produces plane polarised light.

18. (d) : Maximum energy flow in an electromagnetic wave,

$S = E_0 \times B_0 = 100 \times 0.265 = 26.5$ W m^{-2}

19. (b) : $\lambda = 10^{-8}$ m $\rightarrow$ ultraviolet rays,

$\lambda = 10^{-2}$ m $\rightarrow$ microwaves,

$\lambda = 10^8$ m $\rightarrow$ radiowaves

20. (a)

21. (b) : Growth of current in RC circuit

$I_C = I_0 e^{-t/RC})$

Growth of current in LR circuit

$I_L = I_0(1 - e^{-Rt/L})$ $\left[\because\ R = \sqrt{\frac{L}{C}} \text{ or } \frac{R}{L} = \frac{1}{RC}\right]$

$= I_0(1 - e^{-t/RC})$

But $I_C = I_L$

$\therefore\ I_0 e^{-t/RC} = I_0(1 - e^{-t/RC})$

or $2e^{-t/RC} = 1$

or $e^{t/RC} = 2$

or $\frac{t}{RC} = \ln 2$

or $t = RC \ln 2$.

22. (a) : $F_{AB} = k_m\left(\frac{2I_1I_2}{r}\right)l = 30 \times 10^{-7}$ N (attractive)

(as $l = 15$ cm $= 15 \times 10^{-2}$ m, $r = 2$ cm $= 2 \times 10^{-2}$ m)

$F_{CD} = k_m\left(\frac{2I_1I_2}{r'}\right)l = 5 \times 10^{-7}$ N (repulsive)

(as $r' = (10 + 2)$ cm $= 12 \times 10^{-2}$ m)

$F_{net} = F_{AB} - F_{CD} = 25 \times 10^{-7}$ N towards wire

(as $F_{BC} = F_{AD} = 0$)

23. (d) : The radius of the helical path of the electron in the uniform magnetic field is

$r = \frac{mv_\perp}{eB} = \frac{mv\sin\theta}{eB}$

$= \frac{(2.4\times10^{-23}\text{kg m s}^{-1})\times\sin30°}{(1.6\times10^{-19}\text{C})\times(0.15\text{ T})}$

$= 5 \times 10^{-4}$ m $= 0.5 \times 10^{-3}$ m $= 0.5$ mm

24. (b) : Here, $v = 3.2 \times 10^7$ m s^{-1}, $B = 5 \times 10^{-4}$ T

The frequency of electron is

$\upsilon = \frac{eB}{2\pi m_e} = \frac{1.6\times10^{-19}\times5\times10^{-4}}{2\times3.14\times9.1\times10^{-31}}$

$= 1.4 \times 10^7$ Hz $= 14$ MHz

25. (d) : When a ray of light moves from one medium to other, its velocity changes. This change depends on refractive index of the medium. Here, light travels from denser to rarer medium, *i.e.*, from medium of higher refractive index to lower refractive index. So, in rarer medium its velocity increases.

26. (c) : As $I = \frac{200}{\sqrt{R^2 + X_L^2}} = 1$ or $\sqrt{R^2 + X_L^2} = 200$

or $R^2 + X_L^2 = 40000$

$X_L^2 \Rightarrow 40000 - (150)^2$ or $X_L^2 = 17500$

Since $\upsilon' = 100$ Hz and $\upsilon = 50$ Hz, $\therefore\ \upsilon' = 2\upsilon$

As $X_L = 2\pi\upsilon L, \frac{X'_L}{X_L} = \frac{2\pi\upsilon L}{2\pi\upsilon L} = 2$ or $X'_L = 2X_L$

or $X_L'^2 = 4X_L^2 = 4 \times 17500 = 70000$

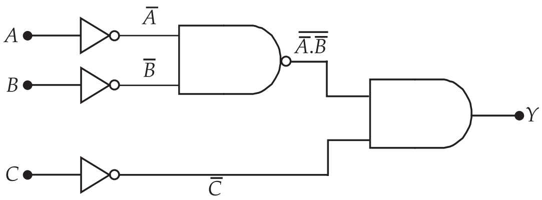
$$Z' = \sqrt{R^2 + X_L'^2} = \sqrt{(150)^2 + 70000}\ \Omega = 304\ \Omega$$

$$I' = \frac{V'}{Z'} = \frac{400\ V}{304\ \Omega} = 1.3\ \text{A}$$

i.e., between 1 A and 2 A

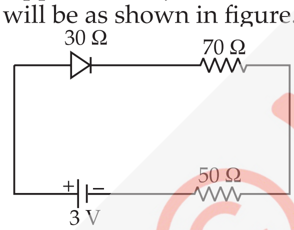
27. **(d)**

28. **(a) :**



$$Y = (\overline{\bar{A}\cdot\bar{B}})\cdot\bar{C} = (\bar{\bar{A}} + \bar{\bar{B}})\bar{C} = (A + B)\bar{C}$$

29. **(c) :** In the circuit the upper diode D_1 is reverse biased and the lower diode D_2 is forward biased. Thus there will be no current across upper diode junction. The effective circuit will be as shown in figure.



Total resistance of circuit

$R = 50 + 70 + 30 = 150\ \Omega$

Current in circuit, $I = \frac{V}{R} = \frac{3\ \text{V}}{150\ \Omega} = 0.02\ \text{A}$

30. **(b) :** Electric field $E = -$ (slope of V-x graph)

and E inside a conductor = 0

$\therefore$ slope of V-x graph between $x = d$ to $x = 2d$ should be zero.

also E in air > E in dielectric

$\therefore$ |slope in air| > |slope in dielectric|

31. **(c) :** $K_1 = \frac{hc}{\lambda_1} - W$ ...(i)

and $K_2 = \frac{hc}{\lambda_2} - W$ ...(ii)

Here, W is work function of given metal surface.

Substituting $\lambda_1 = 2\lambda_2$ in equation (i), we get

$$K_1 = \frac{hc}{2\lambda_2} - W \Rightarrow = \frac{1}{2}\left(\frac{hc}{\lambda_2}\right) - W = \frac{1}{2}(K_2 + W) - W$$

$\therefore$ $K_1 = \frac{K_2}{2} - \frac{W}{2}$ $\left(\because \frac{hc}{\lambda_2} = K_2 + W\right)$

$\Rightarrow$ $K_1 < \frac{K_2}{2}$

32. **(c) :** The torque on the loop must be equal to the gravitational torque exerted about an axis tangent to the loop.

The gravitational torque

$\tau_1 = mgr$ ...(i)

Only B_x causes a torque. Therefore torque to the magnetic field

$\tau_2 = |\vec{M} \times \vec{B}| = MB \sin 90° = \pi r^2 IB_x$ ...(ii)

Hence from equation (i) and (ii), we get

$\tau_1 = \tau_2, \Rightarrow mgr = \pi r^2 IB_x$

$\therefore$ $I = \frac{mg}{\pi r B_x}$

33. **(c) :** $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1}\right)^2 = \frac{9}{1}$

or $\frac{x+1}{x-1} = 3$ $\left(\because x = \sqrt{I_1/I_2}\right)$

$\therefore$ $x = 2$

and $\frac{I_1}{I_2} = x^2 = 2^2 = 4$

Let $I_1 = 4I_2$

$I_2 = I_0 \Rightarrow I_1 = 4I_0$

Since $I_2 = I_1 \cos^2\frac{\delta}{2} \Rightarrow I_0 = 4I_0 \cos^2\frac{\delta}{2}$

$\Rightarrow$ $\cos^2\frac{\delta}{2} = \frac{1}{4}$

or $\cos\frac{\delta}{2} = \frac{1}{2} = \cos\frac{\pi}{3}$ $\therefore$ $\delta = \frac{2\pi}{3}$

and $\delta = \frac{2\pi}{\lambda}\Delta x$ and $\Delta x = \frac{yd}{D}$

$\therefore$ $\left(\frac{2\pi}{\lambda}\right)\left(\frac{yd}{D}\right) = \frac{2\pi}{3} \Rightarrow y = \frac{\lambda D}{3d}$

34. **(c) :** Since refractive index of D layer,

$$\mu = \sqrt{1 - \frac{81.45N}{\upsilon^2}}$$

Here, N = 4000 electrons per c.c.

$= 400 \times 10^6$ electrons per m^3

$\upsilon = 200$ kHz $= 200 \times 10^3$ Hz

$$\therefore \quad \mu = \sqrt{1 - \frac{81.45 \times 400 \times 10^6}{(200 \times 10^3)^2}} = 0.43$$

As $\mu = \frac{c}{v}$

$\therefore$ Phase velocity, $v = \frac{c}{\mu} = \frac{3 \times 10^8}{0.43}$

$= 6.9 \times 10^8$ m s^{-1}

35. **(a) :** As for a telescope magnifying power, $M = \frac{f_0}{f_e}$

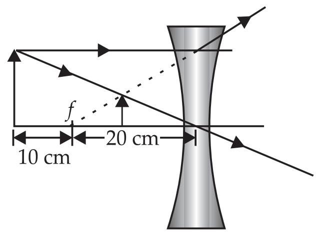
$\therefore \quad 9 = \frac{f_0}{f_e}$ or $f_0 = 9f_e$

Also, $L = f_0 + f_e$ or $20 = f_0 + f_e$

or $\quad 20 = 9f_e + f_e$ or $20 = 10f_e \Rightarrow f_e = 2$ cm

$\therefore \quad f_0 = 9 \times 2$ cm $= 18$ cm

36. **(b) :** When an object is placed between $2f$ and f (focal length) of the diverging lens, the image is virtual, erect and diminished as shown in the ray diagram.



To calculate the distance of the image from the lens, we apply

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{-20} = \frac{1}{v} - \frac{1}{30}$$

$$\Rightarrow \quad v = -\frac{(20)(30)}{20+30}$$

= – 12 cm (to the left of the diverging lens.)

37. **(c) :** The angular momentum is given by

$$L = \sqrt{l(l+1)}\left(\frac{h}{2\pi}\right)$$

For 3d electron, $l = 2$.

$$\therefore \; L = \sqrt{2(3)}\left(\frac{h}{2\pi}\right) = \sqrt{6}\left(\frac{h}{2\pi}\right)$$

38. **(d) :** Helium atom has 2 electrons. When one electron is removed, the remaining atom is hydrogen like atom, whose energy in first orbit is

$E_1 = -(2)^2(13.6 \text{ eV}) = -54.4$ eV

Therefore, to remove the second electron from the atom, the additional energy of 54.4 eV is required. Hence, total energy required to remove both the electrons = 24.6 + 54.4 = 79.0 eV.

39. **(b) :** According to lens maker's formula

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where μ_1 is the refractive index of the medium in which lens is placed and μ_2 is the refractive index of the material of the lens.

$\therefore$ For a convex lens in air

$$\frac{1}{f_a} = \left(\frac{\mu_g}{\mu_a} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(i)}$$

For the same convex lens in liquid

$$\frac{1}{f_l} = \left(\frac{\mu_g}{\mu_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(ii)}$$

Dividing (i) by (ii), we get

$$\frac{f_l}{f_a} = \frac{\mu_g - \mu_a}{\mu_a} \times \frac{\mu_l}{\mu_g - \mu_l}$$

Here, $f_l = 0.150 + 0.225 = 0.375$ m

Substituting the given values, we get

$$\frac{0.375}{0.15} = \frac{\left(\frac{3}{2} - 1\right)}{1} \times \frac{\mu_l}{\left(\frac{3}{2} - \mu_l\right)}$$

$$\Rightarrow \quad \frac{5}{2} = \frac{\mu_l}{3 - 2\mu_l} \quad \text{or} \quad 5(3 - 2\mu_l) = 2\mu_l$$

$12\mu_l = 15$

$$\therefore \quad \mu_l = \frac{15}{12} = \frac{5}{4}$$

40. **(d) :** Let I_0 be the intensity of unpolarized light, then intensity of light from first transparent thin plate of a polaroid is

$$I = \frac{I_0}{2}$$

Now this light will pass through the second similar plate whose axis is inclined at an angle of 30° to that of first plate.

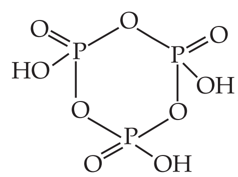
According to Malus law, the intensity of emerging light is

$$I' = I\cos^2 30° = \frac{I_0}{2}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8}I_0$$

$$\therefore \quad \frac{I'}{I_0} = \frac{3}{8}$$

CHEMISTRY

41. **(d) :**

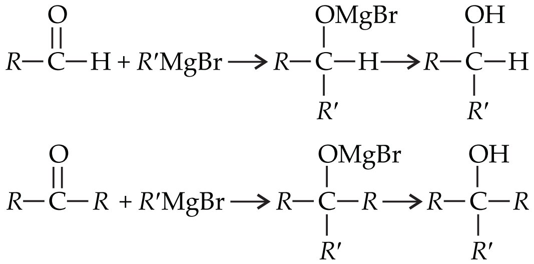


Cyclotrimetaphosphoric acid, $(HPO_3)_3$

42. **(d) :** HCHO will be most polar due to lowest electron density on carbon of carbonyl group.

43. **(b) :** For spontaneity, change in entropy (dS) must be positive, means it should be greater than zero.
Change in Gibbs free energy (dG) must be negative means that it should be lesser than zero. $(dS)_{V,\,E} > 0$, $(dG)_{T,\,P} < 0$.

44. **(b) :** Fehling's solution, Schiff's reagent and Tollens' reagent react only with aldehydes, but Grignard reagent reacts with both aldehydes and ketones.



45. **(b) :** $PbCl_2 \rightleftharpoons Pb^{2+} + 2Cl^-$
$(K_{sp}) = [Pb^{2+}][Cl^-]^2$
$= (6.3 \times 10^{-3}) \times (2 \times 6.3 \times 10^{-3})^2$
$= (6.3 \times 10^{-3}) \times (12.6 \times 10^{-3})^2$

46. **(d) :** Ethanal CH_3CHO, being aldehyde gives positive Fehling's test and it has $CH_3-\overset{O}{\overset{||}{C}}-$ group hence gives positive iodoform test.

47. **(b) :** Maximum number of emission lines

$$= \frac{n(n-1)}{2}$$

$$= \frac{6(6-1)}{2} = \frac{30}{2} = 15 \text{ lines}$$

48. **(b) :** Pseudohalides are monovalent ions made two or more electronegative atoms of which at least one is nitrogen and have properties similar to those of halide ions. The corresponding dimers of pseudohalides are known as pseudohalogens. $RCOO^-$ is not a pseudohalide.

49. **(a) :** Number of radial nodes = $(n - l - 1)$
For 3s: $n = 3$, $l = 0$ (Number of radial node = 2)
For 2p: $n = 2$, $l = 1$ (Number of radial node = 0)

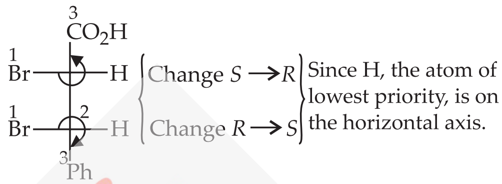
50. **(b) :**

$$CH_3-C\equiv C-CH_2-CH_3 + HBr$$
$$\downarrow$$
$$CH_3-CH=C(Br)-CH_2-CH_3$$
$$+$$
$$CH_3-C(Br)=CH-CH_2CH_3$$

Thus, two structural isomers are formed and each of them exists in a pair of geometrical isomer. Thus, in total 2 structural and 4 configurational (geometrical) isomers are formed.

51. **(a) :** Frenkel defect is not shown by alkali metals because cations and anions have almost same size and cations cannot be accommodated in interstitial sites.

52. **(d) :**



53. **(b) :** Cell reaction is, $Zn + Fe^{2+} \rightarrow Zn^{2+} + Fe$

Using, $E = E° - \frac{0.059}{2} \log \frac{10^{-2}}{10^{-3}}$

($n = 2$, $[Zn^{2+}] = 10^{-2}$ M, $[Fe^{2+}] = 10^{-3}$ M)
Since $E = 0.2905$ V
$\therefore$ $0.2905 = E° - 0.0295$
or $E° = 0.2905 + 0.0295 = 0.32$ V

Again $E° = \frac{0.059}{2} \log K_{eq}$

$\therefore$ $0.32 = \frac{0.059}{2} \log K_{eq}$ or $K_{eq} = 10^{\frac{0.32}{0.0295}}$

54. **(b) :** $X - X'$ bond is weaker than $X - X$ or $X' - X'$ bonds

55. **(c) :** $E_n = Z^2 \times E_1$ for hydrogen - like species.
Threshold energy of $Li^{2+} = 9E_1$
Absorbed energy = Threshold energy + Kinetic energy of photoelectrons

$$E_2 = 9E_1 + \frac{1}{2}mv^2 \Rightarrow mv^2 = 2(E_2 - 9E_1)$$

$$v = \sqrt{\frac{2(E_2 - 9E_1)}{m}}$$

56. **(c)**

57. **(a) :** Glycol serves as an antifreeze for petrol.

58. **(b) :** Equivalent conductivity of any electrolyte at infinite dilution is the sum of the equivalent conductivities of the cation and the anion.

$$\lambda^\circ_{eq} = \lambda^\circ_c + \lambda^\circ_a$$

59. **(b) :** The overall decrease in atomic and ionic radii from La^{3+} to Lu^{3+} is called lanthanoid contraction. Hence, the correct order is
$Yb^{3+} < Pm^{3+} < Ce^{3+} < La^{3+}$.

60. **(d)**

61. **(a) :** The required equation is

$N_2 + 2O_2 \rightleftharpoons N_2O_4;\ K = ?$

The given equations are

$NO_2 \rightleftharpoons \frac{1}{2}N_2 + O_2;\ K_1$ ...(i)

$N_2O_4 \rightleftharpoons 2NO_2;\ K_2$ ...(ii)

Multiply eqn. (i) by 2

$2NO_2 \rightleftharpoons N_2 + 2O_2,\ K' = K_1^2$ ...(iii)

Invert eqn. (ii)

$2NO_2 \rightleftharpoons N_2O_4;\quad K'' = \frac{1}{K_2}$ ...(iv)

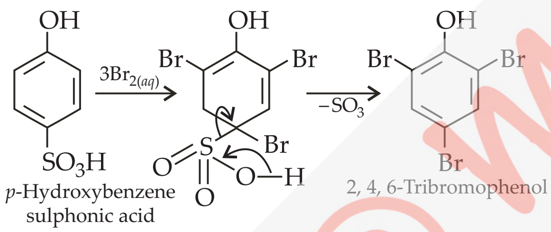
Invert eqn. (iii) and add to eqn. (iv),

$N_2 + 2O_2 \rightleftharpoons 2NO_2;\ \frac{1}{K_1^2}$

$2NO_2 \rightleftharpoons N_2O_4;\ \frac{1}{K_2}$

$$N_2 + 2O_2 \rightleftharpoons N_2O_4;\ \frac{1}{K_1^2} + \frac{1}{K_2}$$

62. **(b) :**



63. **(d) :** Alignment of magnetic moments of antiferromagnetic solids are such that the resultant magnetic moment is zero.

64. **(b) :** Ease of liquefaction of noble gases increases down the group since van der Waals forces of attraction increase down the group with increasing atomic size. Thus, order of ease of liquefaction of noble gases is

Xe > Kr > Ar > Ne > He.

65. **(b) :** (i) represents an electron in 3*s* orbital.

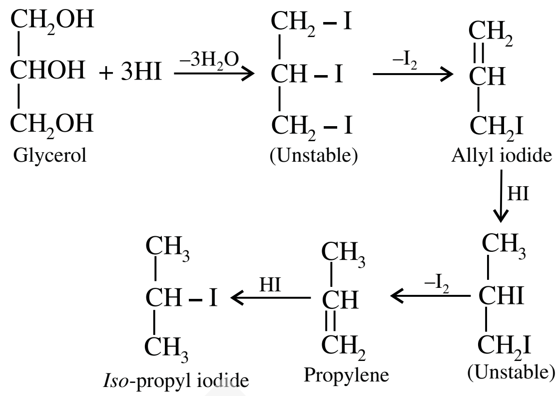
(ii) is not possible as value of *l* must vary from 0, 1, ... $(n-1)$.

(iii) represents an electron in 4*f* orbital.

(iv) is not possible as value of *m* varies from $-l$ ... $+l$.

(v) is not possible as value of *m* varies from $-l$ ... $+l$, it can never be greater than *l*.

66. **(b) :** During the reaction of glycerol with HI, all are formed except $CH_2OH - CHI - CH_2OH$.



67. **(b) :** We know for acidic buffers,

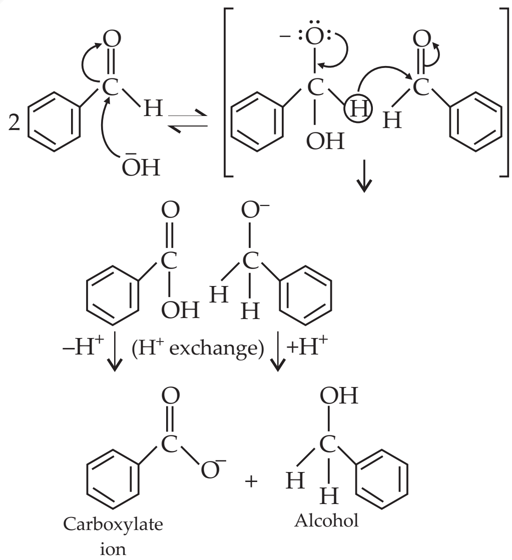
$pH = pK_a + \log\frac{[Salt]}{[Acid]}$

$pK_a = -\log K_a = -\log 10^{-5} = 5$

$\log\frac{[Salt]}{[Acid]} = 6 - 5 = 1$, Taking antilog,

$\frac{[Salt]}{[Acid]} = 10$, *i.e.*, [Salt] : [Acid] = 10 : 1

68. **(a) :** Aldehydes like benzaldehyde, having no α-hydrogen atoms on treatment with NaOH or KOH, are converted to equal amounts of the corresponding carboxylate anion and alcohol *via* Cannizzaro reaction which is proton (H^+)-hydride (H^-) transfer reaction.

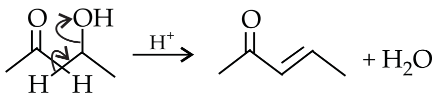


69. **(a) :** It is a general test for carbohydrates. A drop or two of alcoholic solution of α-naphthol is added to 2 mL of glucose solution. 1 mL of conc. H_2SO_4 is added along the sides of the tube and a violet ring is formed at the junction.

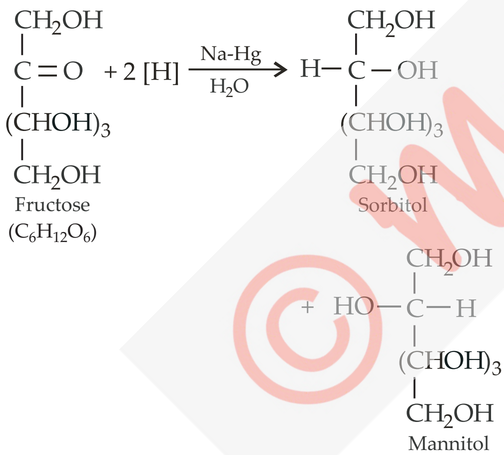
70. **(d) :** According to this law, the total enthalpy change is independent of intermediate steps involved in the change. It depends only on initial and final values of enthalpy change. So it can be used for the calculation of heat of formation, reaction or transition.

71. **(a)**

72. **(a) :** β-Hydroxy ketones or aldols readily undergo dehydration due to the presence of active methylene group.

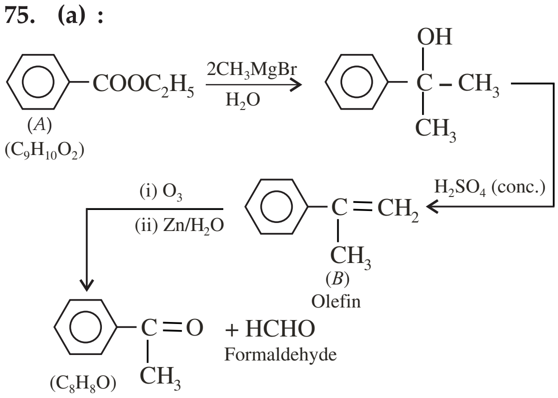


73. **(a) :** Fructose undergoes osazone formation with phenylhydrazine. However, on partial reduction with sodium amalgam and water, a mixture of two epimeric alcohols, sorbitol and mannitol is obtained. This is due to the creation of a new asymmetric carbon atom at C-2.

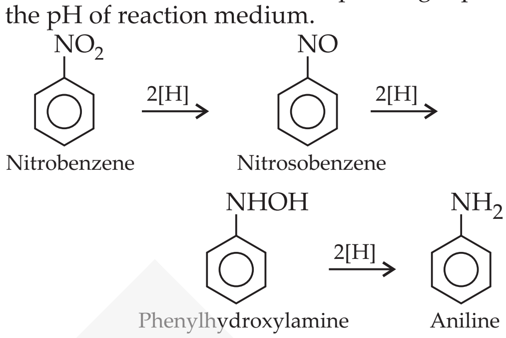


74. **(d) :** For n^{th} order reaction, $t_{1/2} \propto \frac{1}{a^{n-1}}$

75. **(a) :**



76. **(a) :** In the reduction of nitrobenzene, nitrosobenzene ($C_6H_5 - N{=}O$) and phenylhydroxylamines ($C_6H_5 - NHOH$) are obtained as intermediate depending upon the pH of reaction medium.

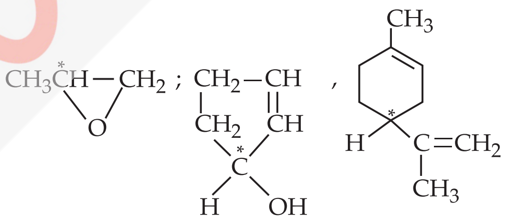


77. **(d) :** SCN^- is a monodentate ligand hence cannot show chelation.

78. **(a) :** $pH = \frac{1}{2}[pK_w + pK_a - pK_b]$

$$pH = 7 + \frac{1}{2}(5.76 - 5.25) = 7.255$$

79. **(c) :** It does not contain chiral carbon.



80. **(d) :** C—Br bond is weaker than C—Cl bond therefore II reacts faster than I and IV. $CH_2{=}CH{-}$ is electron withdrawing while $CH_3CH_2{-}$ is electron donating group, hence nucleophilic attack occurs faster in I than IV *i.e.* I is more reactive than IV. Further, as the reaction follows S_N2 mechanism III is least reactive due to steric hinderance.

MATHEMATICS

81. **(c) :** Let L_1 and L_2 be the given lines, respectively. P is a point on L_1 and Q is a point on L_2. Therefore,

$$P = (t, 2 + 2t, -3 + 3t)$$

and $$Q = (2 + 2s, 6 + 2s, 3 + 4s)$$

where $t, s \in \mathbf{R}$. Now $P = Q$ implies

$$t = 2 + 2s \text{ or } t - 2s = 2 \quad \ldots(i)$$

$$2 + 2t = 6 + 2s \text{ or } 2t - 2s = 4 \quad \ldots(ii)$$

and $$-3 + 3t = 3 + 4s \text{ or } 3t - 4s = 6 \quad \ldots(iii)$$

From Eqs. (i) and (ii) $t = 2$, $s = 0$ which also satisfy Eq. (iii). Therefore the point of intersection is

$$(2, 6, 3) = (x, y, z)$$

so $\therefore y + x = 8$

82. **(a) :** We have

$$[(\vec{a}\times\vec{b})\times\vec{b}]\times\vec{b} = [(\vec{a}\times\vec{b})\cdot\vec{b}]\vec{b} - (\vec{b}\cdot\vec{b})(\vec{a}\times\vec{b})$$
$$= 0(\vec{b}) - 5(\vec{a}\times\vec{b}) \Rightarrow\; = 5(\vec{b}\times\vec{a})$$

83. **(d) :** Let W_j (j = 1, 2, 3, 4) denote 1, 2, 3 and 4 white balls are in the bag. Let W be the ball drawn is white. Then

$$P(W_1) = P(W_2) = P(W_3) = P(W_4) = \frac{1}{4}$$
$$P(W/W_1) = \frac{1}{4} \Rightarrow P(W/W_2) = \frac{2}{4}$$
$$P(W/W_3) = \frac{3}{4} \Rightarrow P(W/W_4) = 1$$

Therefore by Bayes' theorem

$$P(W_4/W) = \frac{P(W_4)P(W/W_4)}{\sum_{j=1}^{4} P(W_j)P(W/W_j)}$$
$$= \frac{\frac{1}{4}\times 1}{\frac{1}{4}\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+\frac{4}{4}\right)} \Rightarrow = \frac{4}{10} = \frac{2}{5}$$

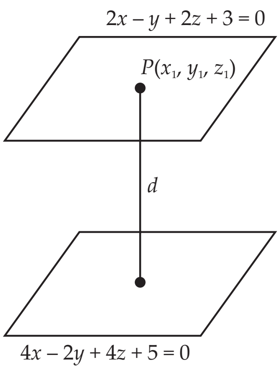
84. **(d) :** We have

Number of even numbers ≤ 100 is equal to 50.
Number of multiples of $3 \le 100$ is 33.
Number of multiples of $5 \le 100$ is 20.
Number of common multiples of 2 and 3 is 16.
Number of common multiples of 3 an 5 is 6.
Number of common multiples of 2 and 5 is 10.
Number of common multiples of 2, 3 and 5 is 3.
Now,

$$P(A) = \frac{50}{100},\; P(B) = \frac{33}{100},\; P(C) = \frac{20}{100}$$
$$P(A\cap B) = \frac{16}{100},\; P(B\cap C) = \frac{6}{100},$$
$$P(C\cap A) = \frac{10}{100}$$

85. **(c) :** Given equations of planes are

$$2x - y + 2z + 3 = 0 \qquad \ldots\text{(i)}$$
and $$4x - 2y + 4z + 5 = 0 \qquad \ldots\text{(ii)}$$



The given planes are parallel as their DR's are proportional.

Let $P(x_1, y_1, z_1)$ be any point lie on the plane (i).

$\therefore\quad 2x_1 - y_1 + 2z_1 + 3 = 0$

$\Rightarrow\quad 2x_1 - y_1 + 2z_1 = -3 \qquad \ldots\text{(iii)}$

$\therefore$ Perpendicular distance d from the point $P(x_1, y_1, z_1)$ to plane (ii) is

$$= \left|\frac{4x_1 - 2y_1 + 4z_1 + 5}{\sqrt{(4)^2+(-2)^2+(4)^2}}\right|$$
$$= \left|\frac{2(2x_1 - y_1 + 2z_1) + 5}{\sqrt{16+4+16}}\right|$$
$$= \left|\frac{2(-3)+5}{\sqrt{36}}\right|$$
$$= \left|\frac{-6+5}{6}\right|$$
$$= \frac{1}{6} \qquad \text{[from Eq. (iii)]}$$

86. **(a) :** Given lines are

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) \qquad \ldots\text{(i)}$$
and $$\vec{r} = 5\hat{i} - 2\hat{j} - \mu(3\hat{i} + 2\hat{j} + 6\hat{k}) \qquad \ldots\text{(ii)}$$

On comparing with $\vec{r} = \vec{a} + \lambda\vec{b}$, we get

$$\vec{b}_1 = \hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b}_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

The angle θ between the two lines is given by

$$\cos\theta = \left|\frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$$
$$= \left|\frac{(\hat{i}+2\hat{j}+2\hat{k})\cdot(3\hat{i}+2\hat{j}+6\hat{k})}{\sqrt{(1)^2+(2)^2+(2)^2}\sqrt{(3)^2+(2)^2+(6)^2}}\right|$$
$$= \left|\frac{(1)(3)+(2)(2)+(2)(6)}{\sqrt{1+4+4}\sqrt{9+4+36}}\right|$$
$$= \left|\frac{3+4+12}{\sqrt{9}\sqrt{49}}\right| = \frac{19}{3\times 7} = \frac{19}{21}$$
$$\therefore\quad \theta = \cos^{-1}\left(\frac{19}{21}\right)$$

87. **(d) :** Given differential equation is

$$\frac{dy}{dx} + \frac{1+\cos 2y}{1-\cos 2x} = 0$$
$$\Rightarrow \frac{dy}{dx} = -\frac{1+\cos 2y}{1-\cos 2x}$$
$$\Rightarrow \frac{dy}{dx} = -\frac{2\cos^2 y}{2\sin^2 x}$$

($\because$ $1 + \cos 2\theta = 2\cos^2\theta$ and $1 - \cos 2\theta = 2\sin^2\theta$)

$$\Rightarrow \frac{dy}{dx} = -\frac{\cos^2 y}{\sin^2 x}$$

On separating the variables, we get

$$\frac{dy}{\cos^2 y} = -\frac{dx}{\sin^2 x}$$

$\sec^2 y\, dy = -\operatorname{cosec}^2 x\, dx$

On integrating both sides, we get

$\int \sec^2 y\, dy = -\int \operatorname{cosec}^2 x\, dx$

$\Rightarrow \tan y = \cot x + C$

which is the required solution.

88. **(b) :** Given differential equation is

$(1 + x^2)dy + 2xy\, dx = \cot x\, dx \qquad (\because\ x \neq 0)$

Above equation can be rewritten as;

$(1 + x^2)dy + (2xy - \cot x)dx = 0$

$\Rightarrow (1 + x^2)dy = (\cot x - 2xy)dx$

On dividing both sides by $1 + x^2$, we get

$$dy = \frac{\cot x - 2xy}{1+x^2}\, dx \Rightarrow \frac{dy}{dx} = \frac{\cot x}{1+x^2} - \frac{2xy}{1+x^2}$$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{\cot x}{1+x^2} \qquad \text{...(i)}$$

This is a linear differential equation of 1st order and is of the form

$$\frac{dy}{dx} + Py = Q \qquad \text{...(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$P = \frac{2x}{1+x^2} \text{ and } Q = \frac{\cot x}{1+x^2}$$

Now, $\text{I.F} = e^{\int P dx} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log|1+x^2|} = 1 + x^2$

$\left[\because\ I_1 = \int \frac{2x}{1+x^2} dx, \text{ put } 1 + x^2 = t \Rightarrow 2x\, dx = dt\right]$

$\Rightarrow \int \frac{dt}{t} = \log|t| + C = \log|1 + x^2| + C]$

Now, solution of linear differential equation is given by,

$y \times \text{I.F} = \int (Q \times \text{I.F})dx$

$$y(1 + x^2) = \int \frac{\cot x}{1+x^2} \times (1 + x^2)dx$$

$\Rightarrow y(1 + x^2) = \int \cot x\, dx$

$\Rightarrow y(1 + x^2) = \log|\sin x| + C$

$(\because\ \int \cot x\, dx = \log|\sin x|)$

$$\Rightarrow y = \frac{\log|\sin x|}{1+x^2} + \frac{C}{1+x^2}$$

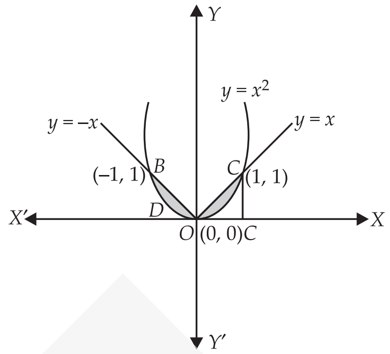
which is the required solution.

89. **(d) :** Given, curve $y = x^2$ ...(i)

and equation of line $y = |x|$ ...(ii)

In first quadrant, the equation of the line is $y = x$ and in second quadrant, the equation of the line is $y = -x$

The points of intersection of the parabola and line are (–1, 1) and (1, 1). As the area to be found is symmetrical about Y-axis.



$\therefore$ Required area = 2(Area of shaded region in the first quadrant)

$= 2\int_0^1 [y_2 \text{ (line)} - y_1\text{(parabola)}]dx$

$= 2\int_0^1 (x - x^2)dx \qquad (\because\ y = x \text{ and } y = x^2)$

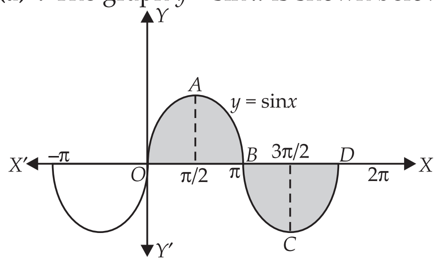
$= 2[\int_0^1 x\, dx - \int_0^1 x^2\, dx]$

$$= 2\left[\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1\right] = 2\left[\left(\frac{1}{2} - 0\right) - \left(\frac{1}{3} - 0\right)\right]$$

$$= 2\left[\frac{1}{2} - \frac{1}{3}\right] = 2\left[\frac{1}{6}\right] = \frac{1}{3} \text{ sq. unit}$$

Therefore, the required area is $\frac{1}{3}$ sq. unit.

90. **(a) :** The graph $y = \sin x$ is shown below.



Required area = Area of region $OABO$ + Area of region $BCDB$

$$= \int_0^{\pi} \sin x\, dx + \left|\int_0^{2\pi} \sin x\, dx\right|$$

(since, in region $BCDB$ the graph is below the X-axis, which area come out to be negative, therefore we take the absolute value)

$= [-\cos x]_0^{\pi} + \left|[-\cos x]_{\pi}^{2\pi}\right|$

$= (-\cos\pi + \cos 0) + |-\cos 2\pi + \cos\pi|$

$= (1 + 1) + |-1 - 1|$

$= 2 + |-2| = 2 + 2 \qquad [\because\ |-2| = -(-2) = 2]$

$= 4$ sq. units

Therefore, the required area is 4 sq. units.

91. (a) : We have

$$f(x) = \cos^{-1}\left(\frac{x^{-1}-x}{x^{-1}+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

and

$$f'(x) = \frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{(1+x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2}$$

$$= \frac{-1}{\sqrt{4x^2}} \cdot \frac{-4x}{(1+x^2)} = \frac{2x}{|x|(1+x^2)}$$

which is an odd function, since $f'(-x) = f'(x)$.

92. (b) : We have

$$y = \cot^{-1}\left(\frac{\ln(e/x^2)}{\ln(ex^2)}\right) + \cot^{-1}\left(\frac{\ln(ex^4)}{\ln(e^2/x^2)}\right)$$

$$= \tan^{-1}\left(\frac{1+\ln x^2}{1-\ln x^2}\right) + \tan^{-1}\left(\frac{2-\ln x^2}{1+2\ln x^2}\right)$$

$$= \tan^{-1} 1 + \tan^{-1}(\ln x^2) + \tan^{-1} 2 - \tan^{-1}(\ln x^2)$$

$$\left[\because\ \tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{A-B}\right)\right]$$

$y = \tan^{-1} 1 + \tan^{-1} 2.$

Hence, $\frac{d^2y}{dx^2} = 0.$

93. (b) : We have, $f_0(x) = x$ and

$$f_n(x) = e^{f_{n-1}(x)} \qquad \ldots(i)$$

Thus, we have

$$f_1(x) = e^x,\ f_2(x) = e^{e^x},\ \ldots\ldots$$

Diferentiating equation (i) w.r.t. x, we have

$$f_n'(x) = f_{n-1}'(x)e^{f_{n-1}(x)} = f_{n-1}'(x)f_n(x)$$

Thus, we have

$f_n'(x) = f_{n-1}'(x)f_n(x)$

$= [f_{n-2}'(x)f_{n-1}(x)]f_n(x)$

$= [f_{n-3}'(x)f_{n-2}(x)]f_{n-1}(x)f_n(x)$

....

....

....

$$= f_0'(x)\, f_1(x) f_2(x) \ldots\ldots f_n(x) = \prod_{r=1}^{n} f_r(x).$$

94. (c) : We have

$$f(x) = \sin\left(\frac{\pi}{x}\right)$$

and $f'(x) = \frac{-\pi}{x^2}\cos\left(\frac{\pi}{x}\right)$

which is negative for value of x, given by

$$\cos\left(\frac{\pi}{x}\right) > 0$$

i.e., $2n\pi - \frac{\pi}{2} < \frac{\pi}{x} < \frac{\pi}{2} + 2n\pi,\ n \in \mathrm{I}$

i.e., $\frac{2}{4n+1} < x < \frac{2}{4n-1},\ n \in \mathrm{I}.$

95. (a, c) : We have

$$y = \ln x,\ x > 1$$

(a) Let $f(x) = y - (x-1) = \ln x - x + 1$

Now, $f'(x) = \frac{1}{x} - 1 < 0\ \forall\ x > 1$

$\Rightarrow$ f is decreasing in $(1, \infty)$

Thus, $f(x) < f(1) = 0 \Rightarrow y < x - 1.$

(b) Let $f(x) = y - \left(1 - \frac{1}{x}\right) = \ln x - 1 + \frac{1}{x}$

Now, $f'(x) = \frac{1}{x} - \frac{1}{x^2} = \frac{x-1}{x^2} > 0\ \forall\ x > 1$

$\Rightarrow$ f is increasing in $(1, \infty)$

Thus, $f(x) > f(1) = 0 \Rightarrow y > 1 - \frac{1}{x}.$

(c) Let $f(x) = y - (x^2 - 1) = \ln x - x^2 + 1$

Now, $f'(x) = \frac{1}{x} - 2x = \frac{-2\left(x^2 - \frac{1}{2}\right)}{x} < 0\ \forall\ x > 1$

$\Rightarrow$ f is decreasing in $(1, \infty)$

Thus, $f(x) < f(1) = 0 \Rightarrow y < x^2 - 1.$

96. (a)

97. (b) : We have

$$f(x) = \frac{x^2}{x^3+200}$$

Then, $f'(x) = \frac{(x^3+200)2x - x^2 \cdot 3x^2}{(x^3+200)^2}$

$$= \frac{-x(x^3-400)}{(x^3+200)^2}$$

From the sign scheme for $f'(x)$, we can see that f attains maxima at $x = (400)^{1/3}$.

−ve | 0 | +ve | $400^{1/3}$ | −ve

Now, we have

$$7 < 400^{1/3} < 8$$

$$a_7 = \frac{7^2}{7^3+200} = \frac{49}{543}$$

and $a_8 = \frac{8^2}{8^3+200} = \frac{64}{712}$

Since $\frac{49}{543} > \frac{64}{712}$, therefore a_7 is the largest term of the given sequence.

98. (a) : Let $I = \int_0^{\pi} [\tan x] dx$

Also, $I = \int_0^{\pi} [\tan(\pi - x)] dx = \int_0^{\pi} [-\tan x] dx$

$= \int_0^{\pi} (-[\tan x] - 1) dx \quad (\because\ [-(I+f)] = -I - 1)$

$= -I - \pi$

gives $I = -\pi/2.$

99. (d) : $\dfrac{1}{1+2\omega}+\dfrac{1}{2+\omega}-\dfrac{1}{1+\omega}$

$=\dfrac{1}{1+2\omega}+\left(\dfrac{1+\omega-2-\omega}{(2+\omega)(1+\omega)}\right)$

$=\dfrac{1}{1+2\omega}+\dfrac{-1}{2+3\omega+\omega^2}$

$=\dfrac{1}{1+2\omega}-\dfrac{1}{1+2\omega}=0$

$(\because\ 2+3\omega+\omega^2=(1+2\omega)+(1+\omega+\omega^2))$

100. (b) : We have

$(2+i)(2+2i)(2+3i)\ \ldots\ldots\ (2+ni)=x+iy$ ...(i)

$\Rightarrow\ \overline{(2+i)(2+2i)(2+3i)\ \ldots\ldots\ (2+ni)}=\overline{x+iy}$

$\Rightarrow\ (2-i)(2-2i)\,(2-3i)\ \ldots\ldots\ (2-ni)=x-iy$ ...(ii)

Multiplying (i) and (ii),we get

$(4-i^2)(4-4i^2)(4-9i^2)\ \ldots\ldots\ (4-n^2i^2)=x^2-i^2y^2$

$\Rightarrow\ 5\,.\,8\,.\,13.\ \ldots\ldots\ (4+n^2)=x^2+y^2.$

101. (b) : Given exp.

$=(1+\omega^2)(1+\omega)+(2+\omega^2)(2+\omega)+(3+\omega^2)(3+\omega)+\ldots\ldots+(n+\omega^2)(n+\omega)$

$=(1+(\omega+\omega^2)+\omega^3)+(4+2(\omega+\omega^2)+\omega^3)+(9+3(\omega+\omega^2)+\omega^3)+\ldots\ldots+(n^2+n(\omega+\omega^2)+\omega^3)$

$=(1-1+1)+(4-2+1)+(9-3+1)+\ldots\ldots+(n^2-n+1)$

$=(1+4+9+\ldots\ldots+n^2)-(1+2+3+\ldots\ldots+n)+n$

$=\dfrac{n(n+1)(2n+1)}{6}-\dfrac{n(n+1)}{2}+n=\dfrac{n(n^2+2)}{3}$

102. (c) : Equation of line through $P(3,4)$ and making angle $\dfrac{\pi}{6}$ with x-axis, is $\dfrac{x-3}{\cos\frac{\pi}{6}}=\dfrac{y-4}{\sin\frac{\pi}{6}}=r$, where r is the distance of any point on the line from $P(3, 4)$.

Coordinates of any point Q on it are

$\left(r\cos\dfrac{\pi}{6}+3, r\sin\dfrac{\pi}{6}+4\right)$

If Q lies on $12x+5y+10=0$, then

$12\left(3+r\dfrac{\sqrt{3}}{2}\right)+5\left(4+\dfrac{r}{2}\right)+10=0$

$\Rightarrow\ r=\dfrac{-132}{12\sqrt{3}+5}$

$\therefore$ Length $PQ=\dfrac{-132}{12\sqrt{3}+5}$.

103. (d) : Clearly, the triangle is right angled as $x-2=0$ and $y-5=0$ are perpendicular.

$\therefore$ The point of intersection (2, 5) is the orthocentre of the triangle.

104. (a) : Let $P(h, k)$ be a point on the circle $15x^2+15y^2-48x+64y=0$. Then the lengths PT_1 and PT_2 of the tangents from $P(h, k)$ to $5x^2+5y^2-24x+32y+75=0$ and $5x^2+5y^2-48x+64y+300=0$ respectively are

$PT_1=\sqrt{h^2+k^2-\dfrac{24}{5}h+\dfrac{32}{5}k+15}$

and $PT_2=\sqrt{h^2+k^2-\dfrac{48}{5}h+\dfrac{64}{5}k+60}$

Since, (h, k) lies on $15x^2+15y^2-48x+64y=0$

$\therefore\ h^2+k^2=\dfrac{48}{15}h-\dfrac{64}{15}k$

$\therefore\ PT_1=\sqrt{\dfrac{48}{15}h-\dfrac{64}{15}k-\dfrac{24}{5}h+\dfrac{32}{5}k+15}$

$=\sqrt{\dfrac{32}{15}k-\dfrac{24}{15}h+15}$

and $PT_2=\sqrt{\dfrac{48}{15}h-\dfrac{64}{15}k-\dfrac{48}{5}h+\dfrac{64}{5}k+60}$

$=2\sqrt{\dfrac{-24}{15}h+\dfrac{32}{15}k+15}=2(PT_1)$

$\therefore\ PT_1 : PT_2 = 1 : 2$

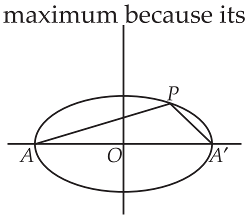
105. (c) : The equation of tangent at $(4,-2)$ to $y^2=x$ is

$-2y=\dfrac{1}{2}(x+4)$ or $x+4y+4=0$

Its slope is $-\dfrac{1}{4}$. Therefore, the slope of the perpendicular line is 4. Since, the tangents at the end of the focal chord of a parabola are at right angle, therefore, the slope of the tangent at Q is 4.

106. (d) : We have $y^2-2y=x-2$ or $(y-1)^2=x-1$. Shifting the origin at (1, 1), we have $x=X+1$, $y=Y+1$. The equation $(y-1)^2=(x-1)$ reduces to $Y^2=X$. This represents a parabola with L.R. = 1. Coordinates of the focus w.r.t. new axes are $(\frac{1}{4}, 0)$. So, the coordinates of the focus w.r.t. old axes are $(\frac{5}{4}, 1)$.

107. (c) : Area of $\Delta APA'$ is maximum when its height is maximum because its base is fixed.



$\therefore$ Maximum area $= \frac{1}{2} \times 2a \times b = ab$.

108. (b) : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, b^2 = a^2(e^2 - 1)$

or $e^2 = 1 + \frac{b^2}{a^2} = \frac{a^2+b^2}{a^2}$

Conjugate hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

or $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1 \therefore e'^2 = 1 + \frac{a^2}{b^2} = \frac{a^2+b^2}{b^2}$

$\therefore \frac{1}{e^2} + \frac{1}{e'^2} = \frac{a^2+b^2}{a^2+b^2} = 1.$

109. (d) : Augmented Matrix $= \begin{bmatrix} 3 & -2 & 1 & 5 \\ 6 & -4 & 2 & 10 \\ 9 & -6 & 3 & 15 \end{bmatrix}$

$\begin{matrix} R_2 - 2R_1 \\ R_3 - 3R_1 \end{matrix} \begin{bmatrix} 3 & -1 & 1 & 5 \\ 0 & -2 & 0 & 0 \\ 0 & -3 & 0 & 0 \end{bmatrix}$

$R_3 - \frac{3}{2}R_2 \begin{bmatrix} 3 & -1 & 1 & 5 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Therefore the given equations have infinitely many solutions.

110. (d) : $A^2 - 4A - 5I = 0 \Rightarrow A(A - 4I) = 5I$

$\Rightarrow A \cdot \frac{1}{5}(A - 4I) = I$

$\Rightarrow A^{-1} = \frac{1}{5}(A - 4I) \quad (\because AA^{-1} = 1)$

111. (a) **112. (a)** **113. (b)** **114. (a)**

115. (b) : Given, $x^2 + y^2 = 1$

On differentiating w.r.t. x, we get

$2x + 2yy' = 0$

$\Rightarrow x + yy' = 0$

Again, differentiating, we get

$1 + yy'' + (y')^2 = 0$

116. (a) : The equation of a member of the family of parabolas having axis parallel to y-axis is $y = Ax^2 + Bx + C$, where A, B and C are arbitrary constant.

$\Rightarrow \frac{dy}{dx} = 2Ax + B \Rightarrow \frac{d^2y}{dx^2} = 2A \Rightarrow \frac{d^3y}{dx^3} = 0$

117. (d) : Given, differential equation can be rewritten as

$\frac{dx}{dy} = \frac{(\log y - x)}{y \log y}$

$\Rightarrow \frac{dx}{dy} + \frac{x}{y \log y} = \frac{1}{y}$

$\therefore$ I.F $= e^{\int \frac{1}{y \log y} dy} = e^{\log(\log y)} = \log y$

118. (d) : Since, the vectors $\vec{a} = 2\hat{i} + \log_3 x\hat{j} + a\hat{k}$ and $\vec{b} = -3\hat{i} + a\log_3 x\hat{j} + \log_3 x\hat{k}$ are inclined at acute angle.

Therefore,

$\vec{a} \cdot \vec{b} > 0$

$\Rightarrow -6 + a(\log_3 x)^2 + a\log_3 x > 0$ for all $x > 0$

$\Rightarrow -6 + ay^2 + ay > 0$, where $y = \log_3 x$

$\Rightarrow ay^2 + ay - 6 > 0$ for all y

$\Rightarrow a > 0$ and $a^2 + 24a < 0$

$\Rightarrow a > 0$ and $a \in (-24, 0)$

But this is not possible. Hence, the vectors $\vec{a}$ and $\vec{b}$ are not inclined at acute angle for any real value of a.

119. (c) : If $\Delta = 0$, then

$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix} = 0$

$\Rightarrow \lambda\vec{a} + \mu\vec{b} + \nu\vec{c} = 0$

(Where λ, μ, ν are consistant)

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are L.D., which is a contradiction.

Hence, Δ can take any non-zero real value.

120. (c) : Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ be the position vector.

The given equation is

$|\vec{r}|^2 + \vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) - 9 = 0$

$\Leftrightarrow$

$|x\hat{i} + y\hat{j} + z\hat{k}|^2 + (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} - \hat{k}) - 9 = 0$

$\Leftrightarrow x^2 + y^2 + z^2 + x + y - z - 9 = 0,$

which is the equation of sphere, whose centre is

$\left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)$

$\therefore$ p.v. of the centre is

$-\frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k} = -\frac{1}{2}(\hat{i} + \hat{j} - \hat{k})$ ·