MOCK TEST PAPER

Full Length Practice Paper

JEE Main Exam Date 6^{th} to 20^{th} April

1. This paper consists of 90 questions.
2. For each correct response 4 marks will be awarded whereas one fourth marks will be deducted for indicating incorrect response of each question.

Time : 3 Hours **Maximum Marks : 360**

PHYSICS

1. The ratio of the kinetic energy required to be given to the satellite to escape earth's gravitational field to the kinetic energy required to be given so that the satellite moves in a circular orbit just above the earth's atmosphere is

(a) one (b) two
(c) half (d) infinity.

2. If E denotes the electric field and ε_0 is the permittivity of free space, the dimensional formula of $\frac{1}{2}\varepsilon_0 E^2$ is

(a) $[ML^2T^{-2}]$ (b) $[ML^2T^{-1}]$
(c) $[ML^{-1}T^{-2}]$ (d) $[MLT^{-2}]$.

3. The density of ice is x g cm^{-3} and that of water is y g cm^{-3}. When m g of ice melts, then the change in volume is

(a) $m(y-x)$ (b) $\frac{y-x}{m}$
(c) $my(x-y)$ (d) $\frac{m}{y}-\frac{m}{x}$.

4. The magnifying power of an astronomical telescope is 15. If the focal length of objective is 90 cm, then the focal length of eye piece is

(a) $\frac{5}{10}$ cm (b) 1 cm
(c) 6 cm (d) 1350 cm.

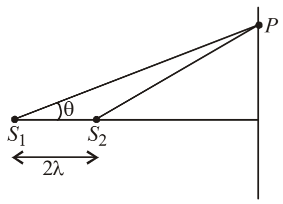
5. A block slides with a velocity of 10 ms^{-1} on a rough horizontal surface. It comes to rest after covering a distance of 50 metre. If g is 10 ms^{-2}, then co-efficient of dynamic friction between the block and the surface is

(a) 0.1 (b) 0.5 (c) 0.6 (d) 1.

6. A ball of radius R carries a positive charge whose volume charge density depends only on the distance r from the ball's centre as : $\rho=\rho_0\left(1-\frac{r}{R}\right)$ where ρ_0 is a constant. Assume ε as the permittivity of the ball. The magnitude of electric field as a function of the distance r inside the ball is given by

(a) $E=\frac{\rho_0}{\varepsilon}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$ (b) $E=\frac{\rho_0}{\varepsilon}\left(\frac{r}{4}-\frac{r^2}{3R}\right)$
(c) $E=\frac{\rho_0}{\varepsilon}\left(\frac{r}{3}+\frac{r^2}{4R}\right)$ (d) $E=\frac{\rho_0}{\varepsilon}\left(\frac{r}{4}+\frac{r^2}{3R}\right)$

7. In Young's double slit experiment, the slits are horizontal. The intensity at a point P shown figure is $\frac{3}{4}I_0$, where I_0 is the maximum intensity. Then the value of θ is, (Given the distance between the two slits S_1 and S_2 is 2λ)



(a) $\cos^{-1}\left(\frac{1}{12}\right)$ (b) $\sin^{-1}\left(\frac{1}{12}\right)$
(c) $\tan^{-1}\left(\frac{1}{12}\right)$ (d) $\sin^{-1}\left(\frac{3}{5}\right)$

8. A surface irradiated with light of wavelength 480 nm gives out electrons with maximum velocity v m/s the cut off wavelength being 600 nm. The same surface would release electrons with maximum velocity $2v$ m/s if it is irradiated by light of wavelength

(a) 325 nm (b) 360 nm
(c) 384 nm (d) 300 nm

9. Light rays of wavelength 6000 Å and of photon intensity 39.6 W/m^2 is incident on a metal surface. If only 1% of photons incident on surface emit photoelectrons, then the number of electrons emitted per second per unit area from the surface will be ($h = 6.64 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m/s)

(a) 12×10^{18} (b) 10×10^{18}
(c) 12×10^{17} (d) 12×10^{16}

10. A tuning fork A is in resonance with an air column 32 cm long and closed at one end. When the length of air column is increased by 1 cm, it is in resonance with another tuning fork B. A and B together give 80 beats in 10 second. The frequency of tuning fork B is

(a) 284 Hz (b) 360 Hz
(c) 384 Hz (d) 256 Hz.

11. The resultant of two resistances connected in parallel is 2 ohm and when connected in series, the resultant becomes 9 ohm. The values of the resistances are

(a) 11 ohm, 7 ohm (b) 7 ohm, 2 ohm
(c) 4.5 ohm each (d) 3 ohm, 6 ohm.

12. A microammeter has a resistance of 100 Ω and a full scale range of 50 μA. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combinations.

(a) 10 mA range with 1 Ω resistance in parallel and 50 volt range with 200 kΩ resistance in series.
(b) 10 volt range with 200 kΩ resistance in series and 5 mA range with 1 Ω resistance in parallel.
(c) 10 volt range with 200 kΩ resistance in series and 10 mA range with 1 Ω resistance in parallel.
(d) 5 mA range with 1 Ω resistance in parallel and 50 volt range with 10 kΩ resistance in series.

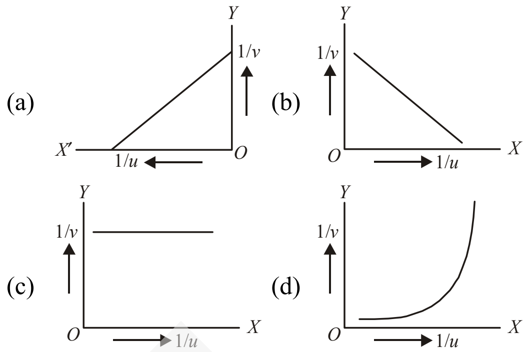
13. A transformer steps up the voltage from 220 V to 11000 volt. If the primary has 100 turns, the secondary should have

(a) 5000 turns (b) 2 turns
(c) 220 turns (d) 11×10^5 turns.

14. A p-n photodiode is made of a material with a band gap of 2.0 eV. The minimum frequency of the radiation that can be absorbed by the material is nearly
(Take hc = 1240 eV nm)

(a) 1×10^{14} Hz (b) 20×10^{14} Hz
(c) 10×10^{14} Hz (d) 5×10^{14} Hz

15. The shape of the graph between $1/u$ and $1/v$ in case of convex lens is



16. A wheel of moment of inertia 5×10^{-3} kg m^2 is making 20 revolutions per second. It is stopped in 20 second. Then angular retardation is

(a) π radian/s^2 (b) 2π radian/s^2
(c) 4π radian/s^2 (d) 8π radian/s^2.

17. The period of revolution of satellite revolving in a circular orbit of radius R is T. The period of revolutions of another satellite in a circular orbit of radius $4R$ is

(a) $T/4$ (b) $T/8$ (c) $4T$ (d) $8T$.

18. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to

(a) $t^{1/2}$ (b) $t^{3/4}$ (c) $t^{3/2}$ (d) t^2.

19. A stone of mass 1 kg tied to a light inextensible string of length $\frac{10}{3}$ m is whirling in a circular path of radius $\frac{10}{3}$ m in a vertical plane. If the ratio of the maximum tension in the string to the minimum tension is 4 and if g is taken to be 10 ms^{-2}, then the speed of the stone at the highest point of the circle is

(a) 20 ms^{-1} (b) $10\sqrt{3}$ ms^{-1}
(c) $5\sqrt{2}$ ms^{-1} (d) 10 ms^{-1}.

20. What is the ratio of the shortest wavelength of the Balmer series to the shortest wavelength of the Lyman series ?

(a) 4 : 1 (b) 4 : 3 (c) 4 : 9 (d) 5 : 9

21. A heavy brass sphere is hung from a spiral spring and it executes vertical vibrations with period T. The ball is now immersed in non-viscous liquid with a density one-tenth that of brass. When set out into vertical vibrations with the sphere remaining inside the liquid all the time, the period will be

(a) $\frac{9}{10}T$ (b) $T\sqrt{\frac{10}{9}}$

(c) unchanged (d) $T\sqrt{\frac{9}{10}}$.

22. A particle moving with a velocity equal to 0.4 ms^{-1} is subjected to an acceleration of 0.15 ms^{-2} for 2 second in a direction at right angle to its direction of motion. The resultant velocity is

(a) 0.7 ms^{-1} (b) 0.5 ms^{-1}

(c) 0.6 ms^{-1}

(d) between 0.7 and 0.1 ms^{-1}.

23. A constant torque acting on a uniform circular wheel changes its angular momentum from J_0 to $4J_0$ in 4 seconds. The magnitude of the torque is

(a) $\frac{3}{4}J_0$ (b) $4J_0$ (c) J_0 (d) $12\ J_0$.

24. The excess pressure inside the first soap bubble is three times that inside the second bubble. Then the ratio of the volumes of the first to second bubble is

(a) 1 : 3 (b) 1 : 9 (c) 1 : 27 (d) 3 : 1.

25. A toy-cart is tied to one end of an unstretched spring of length x. When revolved, the toy-cart moves in a horizontal circle of radius $2x$ with a time period T. When the speed of the toy-cart is so increased that it moves in a horizontal circle of radius $3x$, its time period is T'. The value of T' is

(a) T (b) $\frac{T}{2}$ (c) $\frac{T}{4}$ (d) $\frac{\sqrt{3}}{2}T$.

26. Three capacitors of capacitance 3 μF, 10 μF and 15 μF are connected in series to a voltage source of 100 V. The charge on 15 μF is

(a) 200 μC (b) 100 μC

(c) 25 μC (d) 280 μC.

27. A particle experiences constant acceleration for 6 s after starting from rest. If it travels a distance d_1 in the first two second and a distance d_2 in the next two second and a distance d_3 in the last two second, then

(a) $d_1 : d_2 : d_3 = 1 : 1 : 2$ (b) $d_1 : d_2 : d_3 = 1 : 2 : 3$

(c) $d_1 : d_2 : d_3 = 1 : 3 : 5$ (d) $d_1 : d_2 : d_3 = 1 : 5 : 9$.

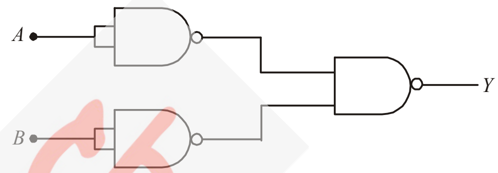
28. The measured mass and volume of a body are 22.42 g and 4.7 cm^3 respectively, with possible errors 0.01 g and 0.1 cm^3. The maximum error in density is about

(a) 0.20% (b) 2.16% (c) 5% (d) 10%.

29. A surface is hit elastically and normally by n balls per unit time, all the balls having the same mass m and moving with the same velocity u. The force on the surface is

(a) mn^2 (b) $2mnu$ (c) $\frac{1}{2}mnu^2$ (d) $2mnu^2$.

30. The given circuit represents



(a) OR gate (b) AND gate

(c) NAND gate (d) NOR gate.

CHEMISTRY

31. 200 g sample of hard water is passed through a cation exchanger in which H^+ ions are exchanged by Ca^{2+} ions. The water coming out of cation exchanger needed 75 mL of 0.1 N NaOH for complete neutralisation. The hardness of water due to Ca^{2+} ion is

(a) 250 ppm (b) 500 ppm

(c) 750 ppm (d) 1000 ppm

32. $R-CH{=}CH-R + X \rightarrow R-\underset{\diagdown\ CH_2\ \diagup}{CH-CH}-R$

The suitable reagent X may be

(a) CH_2-N_2 in light

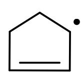
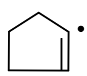
(b) $CH_2{=}C{=}O$ in light

(c) CH_2I_2 in presence of Zn-Cu couple

(d) all of these.

33. Which one is not an allylic free radical?

(a) $CH_2 = CH - \dot{C}H_2$

(b)  (c) 

(d) Both (b) and (c).

34. The degree of dissociation of $Ca(NO_3)_2$ in a dilute solution containing 14 g of the salt per 200 g of water at 100°C is 70%. If the vapour pressure of water is 760 mm, calculate the vapour pressure of solution.

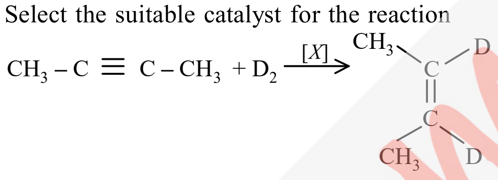
(a) 750.50 mm (b) 745.98 mm
(c) 200.50 mm (d) 14.02 mm

35. Melamine is obtained by the treatment of ammonia with ....
(a) urea (b) cyanuric acid
(c) cyanuryl chloride
(d) amide of cyanuric acid.

36. Which polyhydric phenol is definitely of a ketonic nature?
(a) Hydroquinone (b) Catechol
(c) Phloroglucinol (d) Pyrogallol.

37. Iron exhibits +2 and +3 oxidation states. Which of the following statements about iron is incorrect?
(a) Ferrous oxide is more basic in nature than the ferric oxide.
(b) Ferrous compounds are relatively more ionic than the corresponding ferric compounds.
(c) Ferrous compounds are less volatile than the corresponding ferric compounds.
(d) Ferrous compounds are more easily hydrolysed than the corresponding ferric compounds.

38. Select the suitable catalyst for the reaction



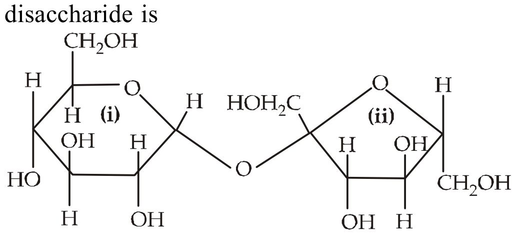
(*cis* - form)

[*X*] may be
(a) Lindlar catalyst (b) Li - ND_3
(c) platinum (d) Na - liquid NH_3.

39. Which one is a zero spin complex?
(a) $[Fe(CN)_6]^{4-}$ (b) $[Ni(CN)_4]^{2-}$
(c) $[Ni(CO)_4]^0$ (d) All of these.

40. A 1.07 mg sample of a compound was dissolved in 78.1 mg of camphor. The solution melted at 176.0ºC. What is the molecular weight of the compound?
(a) 1.6×10^6 (b) 1.6×10^2
(c) 3.2×10^2 (d) 4.8×10^2

41. The correct statement about the following disaccharide is



(a) Ring (i) is pyranose with α-glycosidic linkage.
(b) Ring (i) is furanose with α-glycosidic linkage.
(c) Ring (ii) is furanose with α-glycosidic linkage.
(d) Ring (ii) is pyranose with β-glycosidic linkage.

42. The order of orbital angular momentum quantised is
(a) $3d < 4d < 5d$ (b) $3d > 4d > 5d$
(c) $3d < 4d > 5d$ (d) $3d = 4d = 5d$.

43. The IUPAC name of $[Fe(PPh_3)_3]$ $[Fe(CO)_4]$ is
(a) tris-(tri-phenylphosphine)-iron(II)-tetra-carbonyl- ferrate(II)
(b) tri-phenylphosphine-iron(II)-tetra-carbonylferrate(II)
(c) tris-(triphenylphosphine)iron(II)-tetra-carbonyliron(0)
(d) none of these.

44. Ammonium chloride ionises in liquid NH_3 as

$$NH_4Cl \xrightleftharpoons{\text{liquid } NH_3} NH_4^+ + Cl^-.$$

Thus ammonium chloride acts as .... in liquid NH_3.
(a) acid (b) base
(c) amphoteric (d) salt

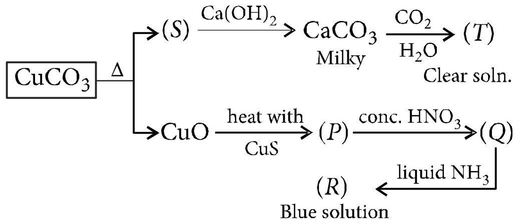
45. The rate constant for the reaction
$2N_2O_5 \rightarrow 4NO_2 + O_2$, is 3.0×10^{-5} sec^{-1}.
If the rate is 2.40×10^{-5} mol litre^{-1}sec^{-1}, then the concentration of N_2O_5 (in mol litre^{-1}) is
(a) 1.4 (b) 1.2 (c) 0.04 (d) 0.8

46. If auric chloride is treated with concentrated ammonia then gold salt .... is obtained which is highly explosive.
(a) purple of cassius
(b) di-amine gold complex ion
(c) fulminating gold
(d) none of these.

47. Match List I with List II and select the correct answer using the codes given below.

List I	List II
(A) Cyanide process	(i) Ultrapure Ge
(B) Floatation process	(ii) Pine oil
(C) Electrolytic reduction	(iii) Extraction of Al
(D) Zone refining	(iv) Extraction of Au

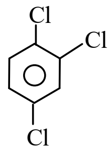
(a) (A)-(iii), (B)-(i), (C)-(iv), (D)-(ii)
(b) (A)-(iv), (B)-(ii), (C)-(iii), (D)-(i)
(c) (A)-(iii), (B)-(ii), (C)-(iv), (D)-(i)
(d) (A)-(ii), (B)-(i), (C)-(iii), (D)-(ii).

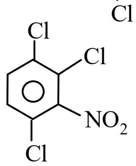
48. 

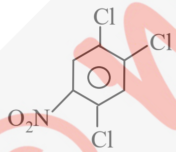
Identify *P*, *R*, *S* and *T*

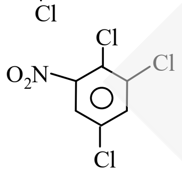
	P	***R***	***S***	***T***
(a)	Cu	CO_2	$Ca(HCO_3)_2$	$[Cu(NO_3)_4]^{2+}$
(b)	Cu_2S	Cu	Cu_2O	$Ca(HCO_3)_2$
(c)	Cu	$[Cu(NH_3)_4]^{2+}$	CO_2	$Ca(HCO_3)_2$
(d)	CO_2	$[Cu(NO_3)_4]^{2+}$	Cu_2O	$Ca(HCO_3)_2$

49. At 500°C, the equilibrium constant for reaction $N_2 + 3H_2 \rightleftharpoons 2NH_3$ is 5.8×10^{-2} litre^2 mole^{-2}. If this equilibrium is attained 5 times faster in presence of catalyst, then the value of K_c at 500°C and in presence of catalyst will be
(a) 58×10^{-2} (b) 0.58×10^{-2}
(c) 580×10^{-2} (d) 5.8×10^{-2}.

50. Nitration of  will give

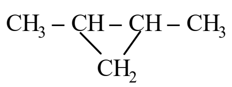
(a) 

(b) 

(c) 

(d) all of these.

51. How many moles of sucrose should be dissolved in 500 g of water so as to get a solution which has a difference of 104°C between boiling point and freezing point (K_f = 1.86 K kg mol^{-1}, K_b = 0.52 K kg mol^{-1})?
(a) 1.68 (b) 3.36 (c) 8.40 (d) 0.840

52. $CH_3 - CH = CH - CH_3 + CH_2I_2 \xrightarrow{\text{Zn-Cu}}$ 

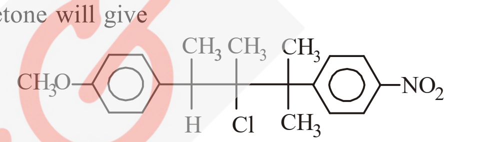
This reaction is called
(a) Diels-Alder reaction
(b) Simmons-Smith reaction
(c) Corey-House reaction
(d) none of these.

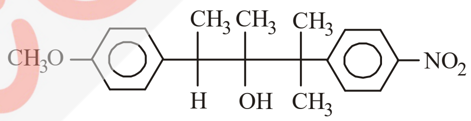
53. The correct order of increasing lattice energy of MgF_2, CaF_2, AgCl and AgBr is
(a) $MgF_2 < CaF_2 < AgCl < AgBr$
(b) $AgBr < AgCl < CaF_2 < MgF_2$
(c) $CaF_2 < AgCl < MgF_2 < AgBr$
(d) $AgBr < AgCl < MgF_2 < CaF_2$.

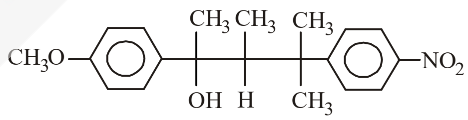
54. Mixing up of equal volumes of 0.1 M NaOH and 0.1 M CH_3COOH yields a solution which is
(a) basic (b) acidic (c) neutral (d) none.

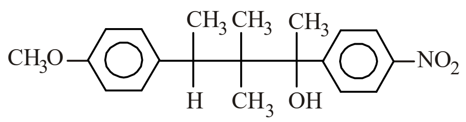
55. Which compound of Xe is stable in solution but explosive, when dry, like T.N.T. ?
(a) XeF_6 (b) XeF_4 (c) XeO_3 (d) $XeOF_4$.

56. The following compound on hydrolysis in aqueous acetone will give



(*K*) 

(*L*) 

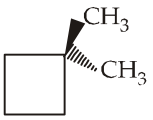
(*M*) 

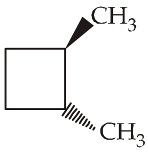
(a) mixture of (*K*) and (*L*)
(b) mixture of (*K*) and (*M*)
(c) only (*M*) (d) only (*K*).

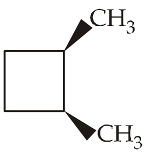
57. The ease of liquefaction of noble gases decreases in the order
(a) He > Ne > Ar > Kr > Xe
(b) Xe > Kr > Ar > Ne > He
(c) Kr > Xe > He > Ar > Ne
(d) Ar > Kr > Xe > He > Ne.

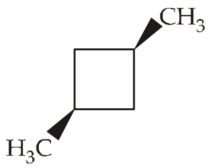
58. The correct order of bond energies of
(i) $H - CH_3$ (ii) $H - CH = CH_2$
(iii) $H - CH_2 - CH = CH_2$
(iv) $H - CH_2 - CH_2 - CH_3$ is
(a) iii > ii > iv > i (b) ii > i > iv > iii
(c) i > ii > iii > iv (d) ii > iii > i > ii.

59. Which of the following is a chiral molecule?

(a) 

(b) 

(c) 

(d) 

60. A gas X at 1 atm is bubbled through a solution containing a mixture of 1 M Y^- and 1 M Z^- at 25°C. If the reduction potential is $Z > Y > X$, then

(a) Y will oxidize X and not Z
(b) Y will oxidize Z and not X
(c) Y will oxidize both X and Z
(d) Y will reduce both X and Z

MATHEMATICS

61. If $aN = \{an : n \in N\}$ and $bN \cap cN = dN$, where $a, b, c \in N$ and b, c are coprime, then

(a) $b = cd$ (b) $c = bd$
(c) $d = bc$ (d) none of these

62. The number of solutions of the equation $1 + \sin x \sin^2 \dfrac{x}{2} = 0$ in $[-\pi, \pi]$ is

(a) zero (b) one (c) two (d) three

63. The equation of straight line through the intersection of the lines $x - 2y = 1$ and $x + 3y = 2$ and parallel to $3x + 4y = 0$, is

(a) $3x + 4y + 5 = 0$ (b) $3x + 4y - 10 = 0$
(c) $3x + 4y - 5 = 0$ (d) $3x + 4y + 6 = 0$

64. The area bounded by $y = \tan^{-1}x$, $x = 1$ and X-axis is

(a) $\left(\dfrac{\pi}{4} + \log\sqrt{2}\right)$ sq. unit

(b) $\left(\dfrac{\pi}{4} - \log\sqrt{2}\right)$ sq. unit

(c) $\left(\dfrac{\pi}{4} - \log\sqrt{2} + 1\right)$ sq. units

(d) none of these

65. The general solution of the differential equation $\dfrac{dy}{dx} + \sin\left(\dfrac{x+y}{2}\right) = \sin\left(\dfrac{x-y}{2}\right)$ is

(a) $\log\tan\left(\dfrac{y}{2}\right) = C - 2\sin x$

(b) $\log\tan\left(\dfrac{y}{4}\right) = C - 2\sin\left(\dfrac{x}{2}\right)$

(c) $\log\tan\left(\dfrac{y}{2} + \dfrac{\pi}{4}\right) = C - 2\sin x$

(d) $\log\tan\left(\dfrac{y}{4} + \dfrac{\pi}{4}\right) = C - 2\sin\left(\dfrac{x}{2}\right)$

66. If $A^2 - A + I = 0$, then inverse of A is

(a) A^{-2} (b) $A + I$ (c) $I - A$ (d) $A - I$.

67. The factors of $\begin{vmatrix} 1 & 1 & 1 \\ x & y & 1 \\ x^2 & y^2 & 1 \end{vmatrix}$ are

(a) $x - 1$, $y - 1$ and $y - x$
(b) $x - 1$, $y - 1$ and x
(c) x, y, and $x - y$
(d) $x - 1$, $y + 1$ and $x + y$.

68. The angle between the vectors $2\hat{i} - 3\hat{j} + \hat{k}$ and $4\hat{i} + \hat{j} - 2\hat{k}$ is given by

(a) $\cos\theta = \dfrac{\sqrt{6}}{41}$ (b) $\cos\theta = \dfrac{\sqrt{6}}{14}$

(c) $\cos\theta = \sqrt{\dfrac{6}{41}}$ (d) none of these.

69. If $f(x) = xe^{x(1-x)}$, then $f(x)$ is

(a) increasing on $\left[-\dfrac{1}{2}, 1\right]$
(b) decreasing on R
(c) increasing on R
(d) decreasing on $\left[-\dfrac{1}{2}, 1\right]$

70. Find x such that the vectors $x\hat{i} + \hat{j} - 2\hat{k}$, $\hat{i} + \hat{j} + 3\hat{k}$ and $8\hat{i} + 5\hat{j}$ are coplanar.

(a) -2 (b) 5 (c) 2 (d) -5.

71. $\tan 100° + \tan 125° + \tan 100° \tan 125° =$

(a) $\sqrt{3}$ (b) -1 (c) $\dfrac{1}{\sqrt{3}}$ (d) 1.

72. A man throws a fair coin a number of times and gets 2 points for each head he throws and 1 point for each tail he throws. The probability that he gets exactly 6 points is

(a) $\frac{21}{32}$ (b) $\frac{23}{32}$ (c) $\frac{41}{64}$ (d) $\frac{43}{64}$

73. $\left|\frac{1}{(2+i)^2} - \frac{1}{(2-i)^2}\right| =$

(a) $\frac{\sqrt{8}}{5}$ (b) $\frac{25}{8}$ (c) $\frac{5}{\sqrt{8}}$ (d) $\frac{8}{25}$.

74. If $x_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$, then $x_1\, x_2\, x_3 \ldots \infty =$

(a) 0 (b) 1 (c) π (d) -1.

75. The equation of the circle described on the line joining the points $(-2, -1)$ and $(3, 4)$ as diameter is

(a) $x^2 + y^2 + x + 3y + 10 = 0$
(b) $x^2 + y^2 - x + 3y + 10 = 0$
(c) $x^2 + y^2 - x - 3y - 10 = 0$
(d) $x^2 + y^2 + x + 3y - 10 = 0$.

76. The radical axis of the circles,
$x^2 + y^2 + 2x + 2y + 1 = 0$ and
$x^2 + y^2 - 10x - 6y + 14 = 0$ is

(a) $4x + 3y - 11 = 0$ (b) $3x - 4y + 11 = 0$
(c) $12x - 8y + 13 = 0$ (d) $12x + 8y - 13 = 0$.

77. t_1 and t_2 are the parameters of the end - points of a focal chord of a parabola. Then

(a) $t_1 + t_2 = -1$ (b) $t_1\, t_2 = -1$
(c) $t_1\, t_2 = 11$ (d) $t_1 + t_2 = 1$.

78. In the ellipse $9x^2 + 5y^2 = 45$, the distance between the foci is

(a) $4\sqrt{5}$ (b) $3\sqrt{5}$ (c) 3 (d) 4.

79. The eccentricity of the hyperbola
$4x^2 - 9y^2 - 8x = 32$ is

(a) $\frac{3}{2}$ (b) $\frac{\sqrt{5}}{3}$ (c) $\frac{\sqrt{13}}{2}$ (d) $\frac{\sqrt{13}}{3}$.

80. The number of integer solutions for the equation $x + y + z + t = 20$ where x, y, z, t are all ≥ -1 is

(a) ${}^{20}C_4$ (b) ${}^{23}C_3$ (c) ${}^{27}C_4$ (d) ${}^{27}C_3$.

81. $\left\{\frac{1+\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}}{1+\cos\frac{\pi}{8} - i\sin\frac{\pi}{8}}\right\}^8 =$

(a) $1 + i$ (b) $1 - i$
(c) 1 (d) -1.

82. A is a matrix of order 3 and $|A| = 8$. Then $|adj\, A| =$

(a) 8 (b) 8^2 (c) 8^3 (d) (1/8).

83. The integer k for which the inequality
$x^2 - 2(4k - 1)x + 15k^2 - 2k - 7 > 0$ is valid for any real x, is

(a) 2 (b) 3
(c) 4 (d) none of these.

84. $\int \frac{dx}{x+\sqrt{x}} =$

(a) $\log(1+\sqrt{x})$ (b) $\log(x+\sqrt{x})$
(c) $2\log(1+\sqrt{x})$ (d) $\frac{2}{2} + \frac{2}{3}x^{3/2}$.

85. $\int_{\pi/4}^{\pi/2} \cot x\, dx =$

(a) log 2 (b) $\log\sqrt{2}$
(c) $\frac{\pi}{2}\log 2$ (d) 2 log 2.

86. If $n > 1$ then $(1 + x)^n - 1 - nx$ is divisible by

(a) x^2 (b) x^5 (c) x^3 (d) x^4.

87. Let R be a relation on a set A such that $R = R^{-1}$ then R is

(a) reflexive (b) symmetric
(c) transitive (d) none of these.

88. The area enclosed between the x - axis and one arc of the curve $y = \sin x$ is

(a) 1 (b) 1/2 (c) 2 (d) π.

89. $\underset{n\to\infty}{Lt}\left(\frac{1}{n} + \frac{1}{\sqrt{n^2-1}} + \frac{1}{\sqrt{n^2-4}} + \ldots\ldots \text{to } n \text{ terms}\right) =$

(a) π (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/4$.

90. The locus of point z satisfying $\mathrm{Re}\left(\frac{1}{z}\right) = k$, where k is a non-zero real number, is

(a) a straight line (b) a circle
(c) an ellipse (d) a hyperbola.

SOLUTIONS

1. **(b) :** $v_e = \sqrt{\frac{2GM}{R}}$ and $v_0 = \sqrt{\frac{GM}{R}}$

The ratio of the two velocities is $\sqrt{2} : 1$. The ratio of the kinetic energies will be 2 : 1.

2. **(c) :** The term $\frac{1}{2}\varepsilon_0 E^2$ represents the energy per unit volume.

So, the dimensional formula is $\frac{[ML^2T^{-2}]}{[L^3]}$

or $[ML^{-1}T^{-2}]$.

3. **(d) :** Volume of m g of ice $= \frac{m}{x}$ and volume of m g of water $= \frac{m}{y}$.

Change in volume $= \frac{m}{y} - \frac{m}{x}$.

4. **(c) :** $M = \frac{f_o}{f_e} \Rightarrow f_e = \frac{f_o}{M} = \frac{90}{15} = 6$ cm.

5. **(a) :** $u = 10$ ms^{-1}, $v = 0$, $S = 50$ m

$v^2 - u^2 = 2aS$; $0^2 - 10^2 = 2a \times 50$;

$a = -1$ ms^{-2}

$$\mu = \frac{f}{R} = \frac{ma}{mg} = \frac{1}{10} = 0.1.$$

6. **(a) :** The given charge distribution in the ball is not uniform but varies w.r.t. distance from the centre. In order to calculate the electric field due to it, the ball can be assumed to be made of various concentric spherical shells. Let us consider one such spherical shell having radius r and thickness dr. Volume of the elementary spherical shell $= 4\pi r^2 dr$

Hence, charge contained in this volume.

$$dq = 4\pi r^2 dr\rho = 4\pi r^2 dr\rho_0\left[1 - \frac{r}{R}\right]$$

$$= 4\pi\rho_0\left[1 - \frac{r}{R}\right]r^2 dr$$

Hence, charge contained within the volume of a sphere of radius $r(r < R)$ is

$$q = \int dq = 4\pi\rho_0 \int_0^r \left[1 - \frac{r}{R}\right]r^2 dr$$

$$= 4\pi\rho_0\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]$$

Now the electric field E at a distance r from the centre of ball can be calculated as if the charge q is concentrated at the centre of the ball.

$$E = \frac{1}{4\pi\varepsilon}\frac{q}{r^2} = \frac{1}{4\pi\varepsilon}4\pi\rho_0\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]\frac{1}{r^2}$$

$$= \frac{\rho_0}{\varepsilon}\left[\frac{r}{3} - \frac{r^2}{4R}\right]$$

7. **(a) :** In Young's double slit experiment, intensity at a point is given by

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

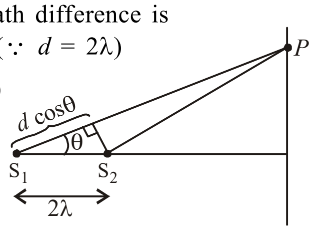
But $\frac{I}{I_0} = \frac{3}{4}$ (given) or $\cos^2\left(\frac{\phi}{2}\right) = \frac{3}{4}$

or $\cos\frac{\phi}{2} = \frac{\sqrt{3}}{2}$ or $\phi = 60° = \frac{\pi}{3}$

Phase difference, $\phi = \frac{2\pi}{\lambda} \times$ path difference

From the figure, path difference is

$d\cos\theta = 2\lambda\cos\theta$ ($\because d = 2\lambda$)



$\therefore \frac{\pi}{3} = \frac{2\pi}{\lambda}\cdot 2\lambda\cos\theta$

$\therefore \cos\theta = \frac{1}{12}$

$\therefore \theta = \cos^{-1}\left(\frac{1}{12}\right)$

8. **(d) :** According to Einstein's photoelectric equation

$$\frac{1}{2}mv_{max}^2 = h\upsilon - h\upsilon_0 \text{ or } \frac{1}{2}mv_{max}^2 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

where λ is the wavelength of incident radiation and λ_0 is threshold wavelength.

$$\therefore \frac{1}{2}mv^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \quad ...(i)$$

$$\frac{1}{2}m(2v)^2 = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda_0}\right) \quad ...(ii)$$

$$\frac{1}{4} = \frac{\frac{1}{\lambda} - \frac{1}{\lambda_0}}{\frac{1}{\lambda'} - \frac{1}{\lambda_0}} \text{ or } \frac{1}{4} = \frac{\frac{1}{480} - \frac{1}{600}}{\frac{1}{\lambda'} - \frac{1}{600}}$$

Solving for λ', we get, $\lambda' = 300$ nm

9. **(c) :** Useful intensity for the emission of electron is

$$I' = 1\%\ I = \frac{1}{100}\times 39.6 = 0.396\ \text{watt/m}^2$$

Energy of each photon $= \dfrac{hc}{\lambda}$

$$= \frac{(6.64\times 10^{-34})\times(3\times 10^{8})}{6000\times 10^{-10}} = 3.32\times 10^{-19}\ \text{J}$$

No. of photoelectrons emitted per second per unit area

$$= \frac{0.396}{3.32\times 10^{-19}} \approx 12\times 10^{17}$$

10. **(d) :** $\upsilon_A - \upsilon_B = \dfrac{80}{10} = 8.$

Also, $\dfrac{v}{4\,l} - \dfrac{v}{4\,l'} = 8,$

l = 32 cm, l' = 33 cm

On simplification, v = 337.92 m s^{-1}.

Now, use $\upsilon_B = \dfrac{v}{4\,l'} = \dfrac{338\times 100}{4\times 33} = 256\ \text{Hz}.$

11. **(d) :** $\dfrac{R_1\,R_2}{R_1 + R_2} = 2\ ;\ R_1 + R_2 = 9$

$\therefore\ R_1\,R_2 = 18\ ;\ R_1 - R_2 = \sqrt{81-72} = 3.$

Adding, $2R_1 = 12,\quad R_1 = 6\ \Omega.$

Again, $R_2 = 9 - R_1 = (9-6)\ \Omega = 3\ \Omega.$

12. **(b) :** For voltmeter, $10 = 50\times 10^{-6}\,(100 + R)$

$\dfrac{10}{50\times 10^{-6}} - 100 = R\ ;$

$0.2\times 10^{6} - 100 = R;\ 2\times 10^{5} - 100 = R$

$R = 200\ \text{k}\Omega$

For ammeter, $1 = \dfrac{50\times 10^{-6}\times 100}{I - 50\times 10^{-6}}$

$I - 50\times 10^{-6} = 50\times 10^{-6}\times 100$

$I = 50\times 10^{-6}(1 + 100)$

$I = 101\times 50\times 10^{-6} \approx 50\times 10^{-4} = 5\ \text{mA}.$

13. **(a) :** $E_p = 220$ V; $E_s = 11000$ V, $N_p = 100$, $N_s = ?$

$$\frac{N_s}{N_p} = \frac{E_s}{E_p}\quad\text{or}\quad N_s = \frac{E_s}{E_p}N_p = \frac{11000}{220}\times 100$$

or $N_s = 5000.$

14. **(d) :** Here, $E_g = 2$ eV

Wavelength of radiation corresponding to this energy is

$$\lambda = \frac{hc}{E_g} = \frac{1240\ \text{eV nm}}{2\ \text{eV}} = 620\ \text{nm}$$

Frequency $\upsilon = \dfrac{c}{\lambda} = \dfrac{3\times 10^{8}\ \text{m/s}}{620\times 10^{-9}\text{m}} = 5\times 10^{14}$ Hz

15. **(a)**

16. **(b) :** υ = 20 rps, t = 20 s, α = ?

$\omega_0 = 2\pi\times 20$ rad s^{-1} = 40π rad s^{-1}

$\omega = 0;\ \ \omega = \omega_0 + \alpha t;\ 0 = 40\pi + 20\alpha$

$\alpha = \dfrac{-40\pi}{20} = -2\pi\ \text{rad s}^{-2}$ or $-\alpha = 2\pi$ rad s^{-2}.

17. **(d) :** $T^2 \propto R^3$ ....(i)

$T'^2 \propto (4R)^3$ ....(ii)

$\dfrac{T'^2}{T^2} = \dfrac{64R^3}{R^3} = 64$ or $\dfrac{T'}{T} = 8$ or $T' = 8T.$

18. **(c) :** Power $= \dfrac{\text{work}}{\text{time}} = \dfrac{\text{force}\times\text{distance}}{\text{time}}$

$$\text{Power} = \frac{\text{mass}\times\text{distance}^2}{\text{time}^3}$$

Since power and mass are constant,

$\therefore\ S^2 \propto t^3$ or $S \propto t^{3/2}.$

19. **(d) :** Maximum tension is at the lowest point. Its value is $\dfrac{m}{r}(v^2 + gr)$ where v is the velocity at the lowest point.

Minimum tension is at the highest point. Its value is $\dfrac{m}{r}(v^2 - 5gr)$. Given $\dfrac{\frac{m}{r}(v^2+gr)}{\frac{m}{r}(v^2-5gr)} = 4$

or $v^2 + gr = 4v^2 - 20gr$

or $3v^2 = 21gr$ or $v^2 = 7gr$

Speed v_H of stone at highest point is $\sqrt{v^2 - 4gr}$.

$\therefore\ v_H = \sqrt{v^2 - 4gr} = \sqrt{7gr - 4gr} = \sqrt{3gr}$

$= \sqrt{3}\times\sqrt{10}\times\sqrt{\dfrac{10}{3}} = 10\ \text{ms}^{-1}.$

20. **(a) :** For a Balmer series

$$\frac{1}{\lambda_B} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \quad ...(i)$$

where n = 3, 4, ...

By putting $n = \infty$ in equation (i), we obtain the series limit of the Balmer series. This is the shortest wavelength of the Balmer series.

or $\lambda_B = \dfrac{4}{R}$ ...(ii)

For a Lyman series

$$\frac{1}{\lambda_L} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right] \quad ...(iii)$$

where $n = 2, 3, 4, ...$

By putting $n = \infty$ in equation (iii), we obtain the series limit of the Lyman series. This is the shortest wavelength of the Lyman series.

or $\quad \lambda_L = \dfrac{1}{R}$ ...(iv)

From equations (ii) and (iv), we get $\dfrac{\lambda_B}{\lambda_L} = \dfrac{4}{1}$

21. (c) : Spring action depends upon the spring constant and is independent of gravity pull or thrust due to liquid.

22. (b) : Velocity acquired in the direction of acceleration

$= 0.15 \times 2 \text{ ms}^{-1} = 0.3 \text{ ms}^{-1}$.

Resultant velocity at the end of 2 s is

$= \sqrt{(0.3)^2 + (0.4)^2}$ or $= 0.5 \text{ ms}^{-1}$.

23. (a) : Change in angular momentum $= 3J_0$

Rate of change of angular momentum $= \dfrac{3J_0}{4}$

This is equal to the applied torque.

24. (c) : $P_1 = 3P_2$; $\dfrac{4T}{r_1} = 3 \times \dfrac{4T}{r_2}$ or $\dfrac{r_1}{r_2} = \dfrac{1}{3}$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}.$$

25. (d) : $F = kx'$ where k is force constant

Also, $F = \dfrac{4\pi^2 m r}{T^2}$

x' = extension, r = total length, x = original length

$\therefore \; kx' = \dfrac{4\pi^2 mr}{T^2}$ or $x' \propto \dfrac{r}{T^2}$ or $T^2 \propto \dfrac{r}{x'}$

In the first case, $T^2 \propto \dfrac{2x}{x}$;

In the second case, $T'^2 \propto \dfrac{3x}{2x}$

$\therefore \; \dfrac{T'^2}{T^2} = \dfrac{3}{2} \times \dfrac{1}{2} = \dfrac{3}{4} \quad \therefore \; T' = \dfrac{\sqrt{3}}{2}T$.

26. (a) : Capacitance of first capacitor (C_1) = 3 μF;
Capacitance of the second capacitor (C_2) = 10 μF;
Capacitance of the third capacitor (C_3) = 15 μF
and the applied potential (V) = 100 V.
The relation for a series combination of the capacitors;

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3} + \frac{1}{10} + \frac{1}{15} = \frac{1}{2}$$

or $C = 2$ μF.

Therefore charge on the 15 μF capacitor (q) = CV
$= 2 \times 100 = 200$ μC.

27. (c) : $S_1 = \dfrac{1}{2}\,a \times 4 = 2a$; $S_2 = \dfrac{1}{2}\,a \times 16 = 8a$;

$S_3 = \dfrac{1}{2}\,a \times 36 = 18a$.

$\therefore \; d_1 : d_2 : d_3 = 2a : 6a : 10a$

or $\; d_1 : d_2 : d_3 = 1 : 3 : 5$.

28. (b) : Density is the ratio of mass and volume. So, maximum percentage error in density is the sum of the maximum percentage error in mass and volume.

Maximum percentage error in mass

$= \dfrac{0.01}{22.42} \times 100 = 0.04\%$

Maximum percentage error in volume

$= \dfrac{0.1}{4.7} \times 100 = \dfrac{100}{47} = 2.12\ \%$.

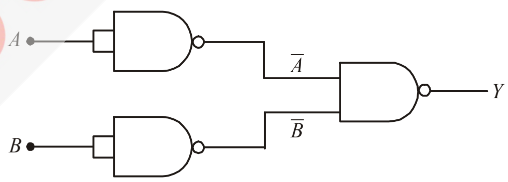
$\therefore$ Maximum percentage error in density
$= 0.04 + 2.12 = 2.16\%$.

29. (b) : Magnitude of change in momentum of one ball is 2 mu. Time taken by one ball is $1/n$.

Rate of change of momentum $= \dfrac{2mu}{1/n} = 2mnu$.

Applying Newton's second law, force on the surface is $2mnu$.

30. (a) :



$Y = \overline{\overline{A} \cdot \overline{B}}$

$Y = \overline{\overline{A}} + \overline{\overline{B}}$ [using $\overline{A \cdot B} = \overline{A} + \overline{B}$]

$Y = A + B$.

Thus, this boolean expression represent OR gate.

31. (c) : m mole of H^+ ion present in 200 g of water coming out of exchanger $= 75 \times 0.1 = 7.5$

$[N_1V_1 = N_2V_2]$

$\therefore$ m mole of Ca^{2+} ion present in hard water $= \dfrac{7.5}{2}$

[1 Ca^{2+} is replaced by $2H^+$ ions.]

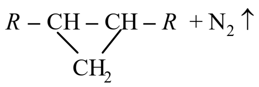
Hence mg of Ca^{2+} ion $= \dfrac{7.5}{2} \times 40 = 150$ mg

$\therefore$ Amount of Ca^{2+} ion present in 200 g of hard water = 150 mg

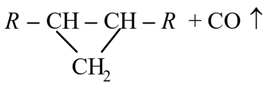
Amount of Ca^{2+} ion present in 10^6 g of hard water

$= \dfrac{150}{200} \times 10^6 \times 10^{-3} = 750$ ppm

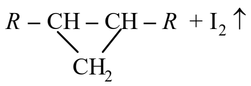
32. (d) : $R-CH=CH-R + CH_2N_2 \xrightarrow{\text{light}}$

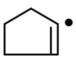


$R-CH=CH-R + CH_2=C=O \xrightarrow{\text{light}}$



$R-CH=CH-R + CH_2-I_2 \xrightarrow[\text{couple}]{\text{Zn - Cu}}$



33. (c) : $CH_2=CH-\dot{C}H_2$ and  are allylic but  is a vinylic free radical.

34. (b) : $\Delta p_{\text{theor.}}$ = Lowering in vapour pressure, when there is no dissociation.

$= p^\circ \times \dfrac{w_2 M_1}{w_1 M_2}$ (Given, $p^\circ = 760$ mm, $w_2 = 14$ g, $w_1 = 200$ g, $M_1 = 18$, $M_2 = 164$)

$= \dfrac{760 \times 14 \times 18}{200 \times 164} = 5.84$ mm

Degree of dissociation $= \dfrac{70}{100} = 0.7$

$\dfrac{\Delta T_{\text{obs.}}}{\Delta T_{\text{theor.}}} = \dfrac{\text{No. of particles after dissociation}}{\text{No. of particles when there is no dissociation}}$

$= \dfrac{1+(n-1)\alpha}{1} = \dfrac{1+(3-1)\times 0.7}{1} = 2.4$

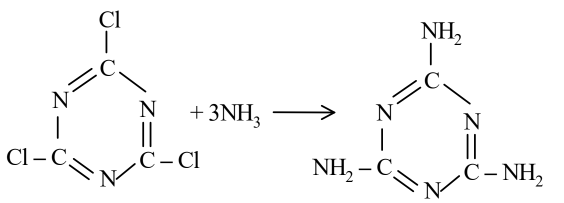
So, $\Delta p_{\text{obs.}} = 2.4 \times \Delta p_{\text{theor.}} = 2.4 \times 5.84$

$= 14.02$ mm

$p^\circ - p_s = \Delta p_{\text{obs.}} = 14.02$

$p_s = p^\circ - 14.02 = 760 - 14.02 = 745.98$ mm

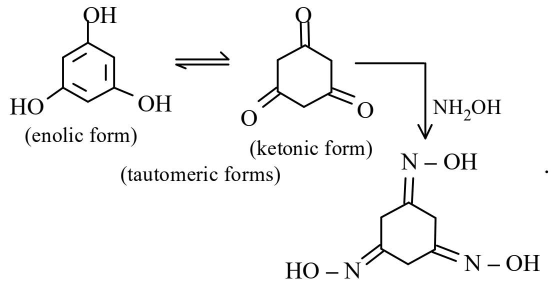
35. (c) :



Cyanuryl chloride (2,4,6-trichloro-1,3,5-triazine)

Melamine (1,3,5-triazine-2,4,6-triamine)

36. (c) :



(Tri-oxime)

37. (d) : (a) FeO > Fe_2O_3 (basic).

(b) $FeCl_2 > FeCl_3$ (ionic), higher the charge, greater the polarizing power and thus, greater the covalent nature.

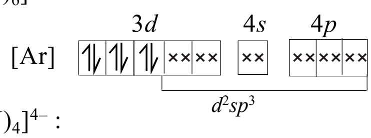
(c) Fe^{2+} salts are more ionic hence, less volatile than Fe^{3+} salts.

(d) Greater the covalent nature, more easily they are hydrolysed. Thus, $FeCl_3$ is more easily hydrolysed than $FeCl_2$.

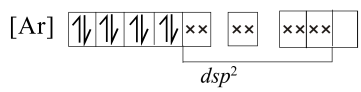
38. (a) : Lindlar's catalyst allows syn-addition.

39. (d) : Magnetic moment = $\sqrt{n(n+2)}$, where n is the number of unpaired electrons. If there is no unpaired electron, then it is called zero spin complex.

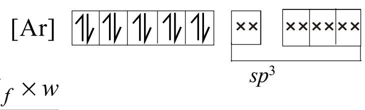
(i) $[Fe(CN)_6]^{4-}$:



(ii) $[Ni(CN)_4]^{4-}$:



(iii) $[Ni(CO)_4]^0$:



40. (b) : $\Delta T_f = \dfrac{K_f \times w}{M \times W}$

$(179.5 - 176.0^\circ C) = \dfrac{40 \times 1.07 \times 10^{-3}}{M \times 78.1 \times 10^{-6}}$

$\therefore \quad M = 1.56 \times 10^2 \approx 1.6 \times 10^2$

41. (a) : The disaccharide is sucrose, with α-glycosidic linkage between C_1 of glucose present in the pyranose form (ring i) and C_2 of fructose present in the furanose form (ring ii).

42. (d) : The orbital angular momentum is given by $\sqrt{l(l+1)}\,\dfrac{h}{2\pi}$, where l is angular momentum quantum number. For d-subshell value of $l = 2$. *i.e.* similar for all d-subshells.

43. (c)

44. (a) : According to solvent system concept acid substance gives rise to a cation characteristic of solvent and base gives rise to an anion characteristic of the solvent.

Liquid NH_3 ionises as $2NH_3 \rightleftharpoons \underset{\text{(acid ion)}}{NH_4^+} + \underset{\text{(base ion)}}{NH_2^-}$.

$$\underset{\text{(Solvent cation)}}{NH_4Cl} \rightleftharpoons NH_4^+ + Cl^-$$

45. (d) : Rate = $k\,[N_2O_5]$

Hence $2.4 \times 10^{-5} = 3.0 \times 10^{-5}\,[N_2O_5]$

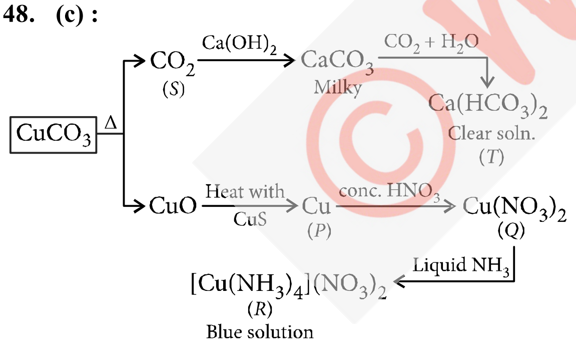
or $\quad [N_2O_5] = \dfrac{2.4\times10^{-5}}{3.0\times10^{-5}} = 0.8$

46. (c) : $AuCl_3 + \underset{\text{(conc.)}}{2NH_3} \rightarrow \underset{\text{(fulminating gold)}}{NH{=}Au - NH_2} + 3HCl$

47. (b) :

Cyanide process	–	Extraction of Au
Floatation process	–	Pine oil
Electrolytic reduction	–	Extraction of Al
Zone refining	–	Ultra pure Ge

48. (c) :



49. (d) : A catalyst does not alter the state of equilibrium.

50. (d) : Due to o- and p-directing –Cl group, all of these products will be formed.

51. (d) : Boiling point of solution

= boiling point + $\Delta T_b = 100 + \Delta T_b$

Freezing point of solution = freezing point – ΔT_f

$= 0 - \Delta T_f$

Difference in temperature (given)

$= 100 + \Delta T_b - (-\Delta T_f)$

$104 = 100 + \Delta T_b + \Delta T_f$

= 100 + molality K_b + molality K_f

= 100 + molality (0.52 + 1.86)

$\therefore \quad$ Molality $= \dfrac{104-100}{2.38} = \dfrac{4}{2.38} = 1.68$ m.

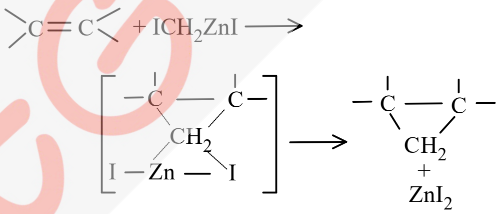
and molality $= \dfrac{\text{moles} \times 1000}{W_{\text{gm (solvent)}}}$;

$1.68 = \dfrac{\text{moles} \times 1000}{500}$

$\therefore \quad$ Moles of solute $= \dfrac{1.68 \times 500}{1000} = 0.84$ mol.

52. (b) : In Simmons-Smith reaction a carbenoid (carbene-like) reagent is an organozinc compound which delivers methylene stereoselectively (and without competing insertion) to the double bond.

$$CH_2I_2 + Zn\text{-}Cu \rightarrow ICH_2ZnI.$$



53. (b) : $AgBr < AgCl < CaF_2 < MgF_2$.

(kJ mol⁻¹) $\quad -883, \quad -895, \quad -2581, \quad -2882.$

As the charge and radii are such that the lattice energy is maximum for MgF_2 and least for AgBr.

L.E. $\propto \dfrac{\text{charge}}{\text{radii}}$

54. (a) : Meq of NaOH = 0.1 V

Meq of CH_3COOH = 0.1 V

$\therefore \quad$ Meq of CH_3COONa formed = 0.1 V

The solution will be alkaline due to hydrolysis of CH_3COONa.

55. (c) : XeO_3 can be detonated by simply rubbing or pressing and produces same effect as T.N.T.

56. (a) : In the given reaction, product K is formed by simple nucleophilic substitution reaction through S_N1 mechanism. Product L is formed as major product by hydride shift, since the carbocation involved in L is stabilised by $-OCH_3$ group.

57. (b) : Ease of liquefaction of noble gases increases down the group since van derwaals forces of attraction increases down the group with increasing atomic size. Thus, order of ease of liquefaction of noble gases is

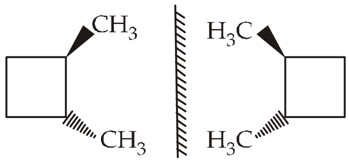
$$Xe > Kr > Ar > Ne > He.$$

58. **(b) :** $H-CH=CH_2 > H-CH_3 >$

$H-CH_2-CH_2-CH_3 > H-CH_2-CH=CH_2$

Bond strength is inversely proportional to the stability of carbocation formed; as allyl carbocation is most stable because of resonance stabilization.

59. **(b) :** A chiral molecule has no superimposable mirror images.



60. **(a) :** Greater the reduction potential, stronger is the oxidizing agent. Hence Y is stronger oxidizing agent than X but weaker than Z.

61. **(c) :** Given, $aN = \{an : n \in N\}$

$\therefore$ $bN = \{bn : n \in N\}$ and $cN = \{cn : n \in N\}$

Also, given $bN \cap cN = dN$

$\therefore$ $bc \in bN \cap cN$ or $bc \in dN$

$\therefore$ $bc = d$ ($\because$ b and c are coprime)

62. **(a) :** Since, $1 + \sin x \sin^2\frac{x}{2} = 0$

$\therefore$ $1+\sin x\left(\frac{1-\cos x}{2}\right)=0$

$\Rightarrow$ $2 + \sin x - \sin x \cos x = 0$

$\Rightarrow$ $\sin 2x - 2\sin x = 4$

which is not possible for any x in $[-\pi, \pi]$.

63. **(c) :** The intersection point of lines $x - 2y = 1$ and $x + 3y = 2$ is $\left(\frac{7}{5}, \frac{1}{5}\right)$.

$\because$ Required line is parallel to $3x + 4y = 0$.

$\therefore$ The slope of required line $= -\frac{3}{4}$

$\therefore$ Equation of required line which passes through $\left(\frac{7}{5}, \frac{1}{5}\right)$ and having slope $-\frac{3}{4}$, is

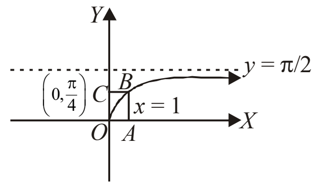
$\Rightarrow$ $y-\frac{1}{5}=\frac{-3}{4}\left(x-\frac{7}{5}\right) \Rightarrow \frac{3x}{4}+y=\frac{21}{20}+\frac{1}{5}$

$\Rightarrow$ $\frac{3x+4y}{4}=\frac{21+4}{20} \Rightarrow 3x + 4y = 5$

$\Rightarrow$ $3x + 4y - 5 = 0$

64. **(b) :** Required area

= Area of rectangle $OABC$ – Area of curve $OBCO$



$$= \frac{\pi}{4} - \int_0^{\pi/4} \tan y \, dy = \frac{\pi}{4} + [\log \cos y]_0^{\pi/4}$$

$$= \frac{\pi}{4} + \log\cos\frac{\pi}{4} - \log\cos(0)$$

$$= \frac{\pi}{4} + \log 1 - \log\sqrt{2} - \log 1 = \left(\frac{\pi}{4} - \log\sqrt{2}\right) \text{ sq. unit}$$

65. **(b) :** Given equation

$$\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \sin\left(\frac{x-y}{2}\right) - \sin\left(\frac{x+y}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = -2\sin\left(\frac{y}{2}\right)\cos\left(\frac{x}{2}\right)$$

$$\Rightarrow \operatorname{cosec}\left(\frac{y}{2}\right)dy = -2\cos\left(\frac{x}{2}\right)dx$$

On intergrating both sides, we get

$$\int \operatorname{cosec}\left(\frac{y}{2}\right)dy = -\int 2\cos\left(\frac{x}{2}\right)dx + C_1$$

$$\Rightarrow \frac{\log\left(\tan\frac{y}{4}\right)}{\frac{1}{2}} = -\frac{2\sin\left(\frac{x}{2}\right)}{\frac{1}{2}} + C_1$$

$$\Rightarrow \log\left(\tan\frac{y}{4}\right) = C - 2\sin\left(\frac{x}{2}\right)$$

66. **(c) :** $A^2 - A + I = 0$.

Multiplying by A^{-1}, we get

$A^2A^{-1} - A\,A^{-1} + A^{-1} = 0$

$\Rightarrow$ $A\,I - I + A^{-1} = 0$ ($\because A\,A^{-1} = I$)

$\Rightarrow$ $A - I + A^{-1} = 0$ ($\because$ $A\,I = A$)

$\Rightarrow$ $A^{-1} = I - A$.

67. **(a) :** $\begin{vmatrix} 1 & 1 & 1 \\ x & y & 1 \\ x^2 & y^2 & 1 \end{vmatrix}$

Applying $C_1 \to C_1 - C_3;\ \ C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & 1 \\ x-1 & y-1 & 1 \\ x^2-1 & y^2-1 & 1 \end{vmatrix}$$

$$= (x-1)(y-1) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ x+1 & y+1 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$

$$= (x-1)(y-1) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ x-y & y+1 & 1 \end{vmatrix}$$

$= (x-1)(y-1)(y-x)$.

68. **(b) :** $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\ |\vec{b}|} = \dfrac{8-3-2}{\sqrt{4+9+1}\ \sqrt{16+1+4}}$

$$= \frac{3}{\sqrt{14}\ \sqrt{21}} = \frac{\sqrt{3}\ \sqrt{3}}{\sqrt{2}\ \sqrt{7}\ \sqrt{3}\ \sqrt{7}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}\ \times\ 7} = \frac{\sqrt{3}\ \sqrt{2}}{7(2)} = \frac{\sqrt{6}}{14}.$$

69. **(a) :** $f(x) = xe^{x(1-x)}$

On differentiating w.r.t. x, we get

$$f'(x) = e^{x(1-x)} + x\cdot e^{x(1-x)}\cdot(1-2x)$$
$$= e^{x(1-x)}\cdot\{1 + x(1-2x)\}$$
$$= e^{x(1-x)}\cdot(-2x^2 + x + 1)$$

It is clear that $e^{x(1-x)} > 0$ for all x.

Now, by sign rule for $-2x^2 + x + 1$

$f'(x) \ge 0$, if $x \in \left[-\dfrac{1}{2}, 1\right]$

so, $f(x)$ is increasing on $\left[-\dfrac{1}{2}, 1\right]$.

70. **(c) :** Given vectors are coplanar

$$\therefore \quad \begin{vmatrix} x & 1 & -2 \\ 1 & 1 & 3 \\ 8 & 5 & 0 \end{vmatrix} = 0$$

i.e. $x(0-15) - 1(0-24) - 2(5-8) = 0$

i.e. $-15x + 24 + 6 = 0 \qquad \therefore \quad x = 2.$

71. **(d) :** $\tan 225° = \tan(180° + 45°) = \tan 45° = 1$

i.e. $\tan(100° + 125°) = 1$;

$$\frac{\tan 100° + \tan 125°}{1 - \tan 100°\ \tan 125°} = 1$$

$\therefore \quad \tan 100° + \tan 125° = 1 - \tan 100° \tan 125°$

i.e. $\tan 100° + \tan 125° + \tan 100° \tan 125° = 1.$

72. **(d) :** $P(HHH) + P(HHTT) + P(HTTTT) + P(TTTTTT)$

$$= \frac{1}{2^3} + \left({}^4C_2\right)\frac{1}{2^4} + \left({}^5C_1\right)\frac{1}{2^5} + \frac{1}{2^6} = \frac{43}{64}$$

73. **(d) :** $\left|\dfrac{1}{(2+i)^2} - \dfrac{1}{(2-i)^2}\right| = \left|\dfrac{(2-i)^2 - (2+i)^2}{(2+i)^2\ (2-i)^2}\right|$

$$= \left|\frac{-8i}{25}\right| = \frac{8}{25}.$$

74. **(d) :** $x_1 x_2 x_3 \ldots . \infty$

$$= \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{2^2} + i\sin\frac{\pi}{2^2}\right)\left(\cos\frac{\pi}{2^3} + i\sin\frac{\pi}{2^3}\right)\ldots$$

$$= \cos\left(\frac{\pi}{2} + \frac{\pi}{2^2} + \frac{\pi}{2^3} + \ldots.\text{ to } \infty\right) + i\sin\left(\frac{\pi}{2} + \frac{\pi}{2^2} + \frac{\pi}{2^3}\ldots\text{to } \infty\right)$$

$$= \cos\frac{\pi}{2}\left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots.\text{ to } \infty\right) + i\sin\frac{\pi}{2}\left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3}\ldots\text{ to } \infty\right)$$

$$= \cos\frac{\pi}{2}(2) + i\sin\frac{\pi}{2}(2)$$

$$\left[\because\ 1 + \frac{1}{2} + \frac{1}{2^2} + \ldots\text{ to } \infty = 2\right]$$

$= \cos\pi + i\sin\pi = -1.$

75. **(c) :** The equation of the circle is

$(x+2)(x-3) + (y+1)(y-4) = 0$

i.e. $x^2 + y^2 - x - 3y - 10 = 0.$

76. **(d) :** The radical axis is $S_1 - S_2 = 0$,

i.e. $12x + 8y - 13 = 0.$

77. **(b) :** $P \equiv (at_1^2, 2at_1)$, $Q \equiv (at_2^2, 2at_2)$

Equation of the chord PQ is $\dfrac{y - 2at_1}{x - at_1^2} = \dfrac{2at_2 - 2at_1}{at_2^2 - at_1^2}$

i.e., $\dfrac{y - 2at_1}{x - at_1^2} = \dfrac{2}{t_1 + t_2}.$

This passes through the focus $S \equiv (a, 0)$

$\therefore$ put $x = a$ and $y = 0$.

$$\therefore \quad \frac{0 - 2at_1}{a - at_1^2} = \frac{2}{t_1 + t_2}$$

i.e., $-2at_1^2 - 2at_1t_2 = 2a - 2at_1^2 \quad$ *i.e.* $t_1 t_2 = -1.$

78. **(d) :** $\dfrac{x^2}{5}+\dfrac{y^2}{9}=1$; $a^2 = 5, b^2 = 9,\ a < b$

$e^2 = 1-\dfrac{a^2}{b^2} = 1 - \dfrac{5}{9} = \dfrac{4}{9}$. $\quad \therefore\ e = \dfrac{2}{3}$.

Distance between the foci $= 2\,b\,e = 2 \times 3 \times \dfrac{2}{3} = 4$.

79. **(d) :** $4x^2 - 9y^2 - 8x = 32$

i.e. $4(x^2 - 2x) - 9y^2 = 32$

i.e. $4[(x-1)^2 - 1] - 9y^2 = 32$

$\therefore\ 4(x-1)^2 - 9y^2 = 36 \Rightarrow \dfrac{(x-1)^2}{9} - \dfrac{y^2}{4} = 1$

$a^2 = 9,\ b^2 = 4$;

$e^2 = 1 + \dfrac{b^2}{a^2} = 1 + \dfrac{4}{9} = \dfrac{13}{9} \quad \therefore\ e = \dfrac{\sqrt{13}}{3}$.

80. **(d) :** Let $u = x + 1,\ v = y + 1,\ w = z + 1$ and $p = t + 1$

$\therefore\ u, v, w, p \ge 0$ and $u + v + w + p = 24$.

So, required number of solutions is

${}^{24+4-1}C_{4-1} = {}^{27}C_3$.

81. **(d) :** Put $Z = \left(\cos\dfrac{\pi}{8} + i\sin\dfrac{\pi}{8}\right)$

then the given expression $= \left(\dfrac{1+Z}{1+\dfrac{1}{Z}}\right)^8$

$= Z^8 = \cos\pi + i\sin\pi = -1$.

82. **(b) :** We know that $A \cdot adj\,A = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$

$\therefore\ |A| \cdot |adj\,A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$

i.e. $|A|\,|adj\,A| = |A|^3$

$\therefore\ |adj\,A| = |A|^2 = 8^2$. $\quad (\because |A| = 8)$

83. **(b) :** Let $f(x) = x^2 - 2(4k-1)x + 15k^2 - 2k - 7$, then

$f(x) > 0 \Rightarrow D < 0\ (\because$ coeff. of $x^2 > 0)$

$\Rightarrow\ 4(4k-1)^2 - 4(15k^2 - 2k - 7) < 0$

$\Rightarrow\ k^2 - 6k + 8 < 0 \Rightarrow 2 < k < 4$.

84. **(c) :** $\displaystyle\int\frac{1}{x+\sqrt{x}}dx = \int\frac{1}{\sqrt{x}\,(\sqrt{x}+1)}dx = 2\int\frac{1}{t}\,dt$

$= 2\log t = 2\log(\sqrt{x}+1)$.

85. **(b) :** $\displaystyle\int_{\pi/4}^{\pi/2} \cot x\,dx = [\log\ \sin x]_{\pi/4}^{\pi/2}$

$= \log\sin\dfrac{\pi}{2} - \log\sin\dfrac{\pi}{4}$

$= \log 1 - \log\left(\dfrac{1}{\sqrt{2}}\right) = \log\sqrt{2}$

86. **(a) :** For $n > 1$, we have

$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \ldots\ {}^nC_nx^n$

$\Rightarrow\ (1+x)^n = 1 + nx + ({}^nC_2x^2 + {}^nC_3x^3 + \ldots\ {}^nC_nx^n)$

$\Rightarrow\ (1+x)^n - 1 - nx = x^2({}^nC_2 + {}^nC_3x + {}^nC_4x^2 + \ldots.\ {}^nC_nx^{n-2})$

Clearly, R.H.S. is divisible by x^2 so is L.H.S.

87. **(b) :** Let $(a, b) \in R$.

Then $(a, b) \in R$

$\Rightarrow\ (b, a) \in R^{-1}$ [by def. of R^{-1}]

$\Rightarrow\ (b, a) \in R$ $[\because R = R^{-1}]$

So, R is symmetric.

88. **(c) :** Area $= \displaystyle\int_0^{\pi} \sin x\,dx = [-\cos x]_0^{\pi}$

$= (-\cos\pi) - (-\cos 0) = 2$.

89. **(b) :** $\displaystyle\underset{n\to\infty}{Lt}\left(\frac{1}{n}+\frac{1}{\sqrt{n^2-1}}+\frac{1}{\sqrt{n^2-4}}+\ldots.\text{ to } n \text{ terms}\right)$

$\displaystyle= \underset{n\to\infty}{Lt}\sum_{r=0}^{n-1}\frac{1}{\sqrt{n^2-r^2}}$

$\displaystyle= \underset{n\to\infty}{Lt}\ \frac{1}{n}\sum_{r=0}^{n-1}\frac{1}{\sqrt{1-\left(\frac{r}{n}\right)^2}} = \int_0^1\frac{dx}{\sqrt{1-x^2}}$

$\left[\because f\left(\dfrac{r}{n}\right) = \dfrac{1}{\sqrt{1-(r/n)^2}},\ f(x) = \dfrac{1}{\sqrt{1-x^2}}\right]$

$= [\sin^{-1}x]_0^1 = \sin^{-1}(1) - \sin^{-1}(0) = \dfrac{\pi}{2}$.

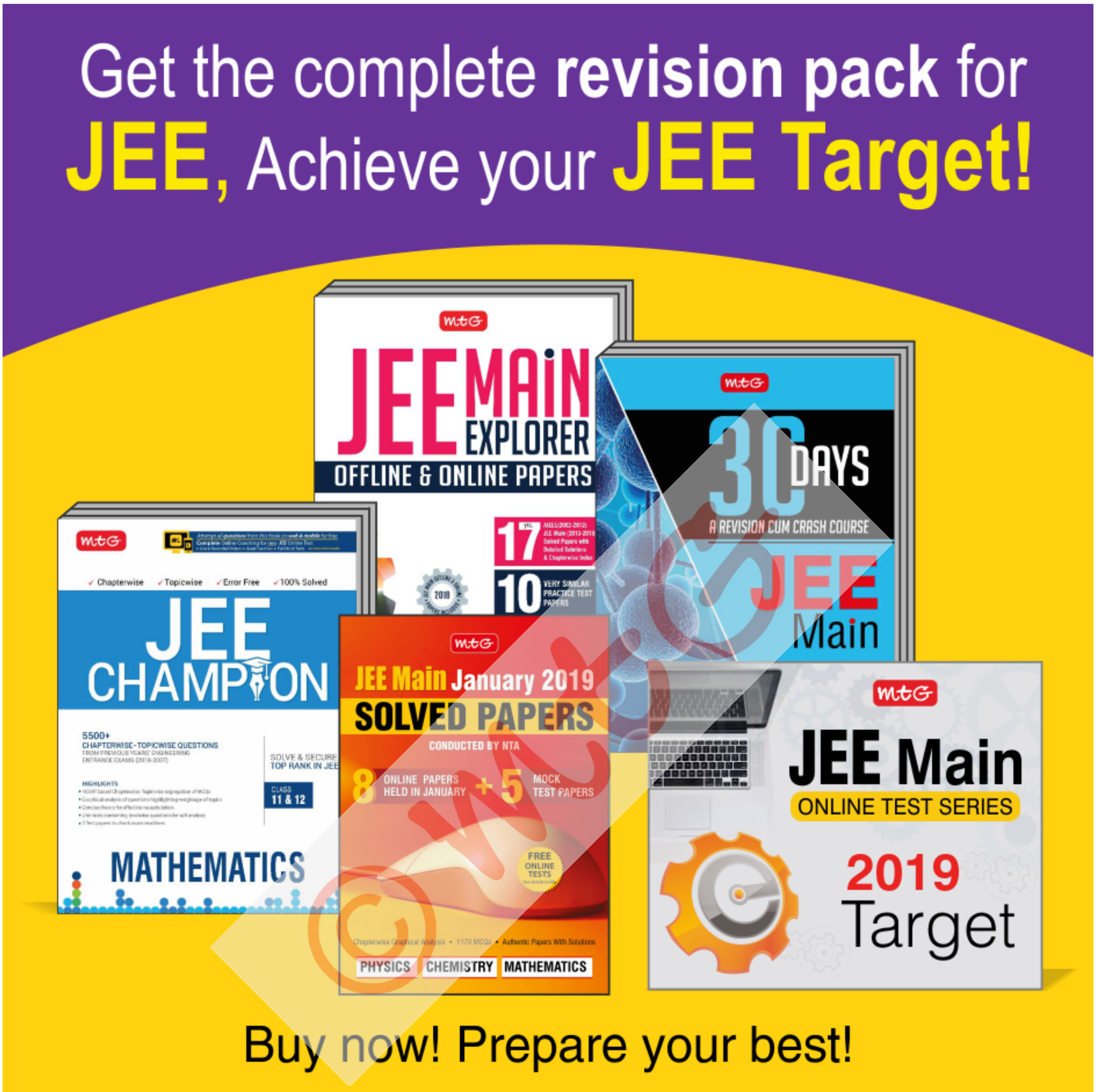
90. **(b) :** Let $z = x + iy$

Then $\text{Re}\left(\dfrac{1}{z}\right) = k \Rightarrow \text{Re}\left(\dfrac{1}{x+iy}\right) = k$

$\Rightarrow\ \text{Re}\left(\dfrac{x}{x^2+y^2} - \dfrac{iy}{x^2+y^2}\right) = k$

$\Rightarrow\ \dfrac{x}{x^2+y^2} = k \Rightarrow x^2 + y^2 - \dfrac{1}{k}x = 0$

which is the equation of a circle.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