EXAM DATE
8^{th}, 9^{th}, 10^{th} & 12^{th} April

Mathematics Practice Problems

JEE Main

- Continuity and Differentiability
- Application of Derivatives
- Integrals
- Application of Integrals

Time: 100 min. **Max. Marks:** 200

LEVEL - 1

1. If $f(x)=\begin{cases}\dfrac{\sqrt{1+\lambda x}-\sqrt{1-\lambda x}}{x}, & -1\le x<0\\ \dfrac{|2x+1|}{x-1}, & 0\le x\le 1\end{cases}$ is continuous in [–1, 1], then λ equals

(a) –1/2 (b) –1 (c) 1/2 (d) 1

2. The radius of a circle is increasing at the rate of 0.1 cm/sec. When the radius of the circle is 5 cm, the rate of change of its area is

(a) π cm^2/sec (b) 10π cm^2/sec
(c) 0.1π cm^2/sec (d) 5π cm^2/sec

3. Let $y' = e^{-2x}$ and $y = 0$ when $x = e$. Then the value of x when $y=\dfrac{1}{2}$ is

(a) $e-1$ (b) $\dfrac{1}{2}(e-1)$
(c) $\dfrac{1}{2}(3e-1)$ (d) None of these

4. The area bounded by the curve $\sqrt{x}+\sqrt{y}=1$ and the coordinate axes is

(a) 1 (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{1}{6}$

5. If $g(x)$ is a differentiable function such that $g(0) = g(1) = 0$, $g'(1) = 1$ and $y(x) = g(e^x)\cdot e^{g(x)}$, then $y'(0)$ equals

(a) 0 (b) 1 (c) 2 (d) e

6. The equation of normal to the curve $y = (1 + x)^y + \sin^{-1}(\sin^2 x)$ at $x = 0$ is

(a) $x + y = 1$ (b) $x - y = 1$
(c) $x + y = -1$ (d) $x - y = -1$

7. $\displaystyle\int \frac{dx}{x(x^7+1)} =$

(a) $\log_e\left(\dfrac{x^7}{x^7+1}\right)+c$ (b) $\dfrac{1}{7}\log_e\left(\dfrac{x^7}{x^7+1}\right)+c$

(c) $\log_e\left(\dfrac{x^7+1}{x^7}\right)+c$ (d) $\dfrac{1}{7}\log_e\left(\dfrac{x^7+1}{x^7}\right)+c$

8. The area bounded by the curves $y = ax^2$ and $x = ay^2$ is equal to 1. Then $a =$

(a) $\dfrac{1}{\sqrt{3}}$ (b) $\dfrac{1}{2}$ (c) 1 (d) $\dfrac{1}{3}$

9. If $y=\sin^{-1}(3^{-x})$, then $\dfrac{dy}{dx}=$

(a) $-\dfrac{\log 3}{\sqrt{3^{2x}-1}}$ (b) $\dfrac{3^x\log 3}{\sqrt{3^{2x}-1}}$

(c) $\dfrac{-3^x\log 3}{\sqrt{3^{2x}-1}}$ (d) $\dfrac{\log 3}{3^x\sqrt{3^{2x}-1}}$

10. A point on the curve $2y^3 + x^2 = 12y$ at which the tangent to the curve is vertical is

(a) $(\sqrt{2},\sqrt[4]{128})$ (b) $(\sqrt[4]{128},\sqrt{2})$
(c) $(2,\sqrt[4]{128})$ (d) $(\sqrt[4]{128},2)$

11. If $I_1=\displaystyle\int_0^{\pi/2} x\sin x\,dx$ and $I_2=\displaystyle\int_0^{\pi/2} x\cos x\,dx$, then which one of the following is true?

(a) $I_1+I_2=\dfrac{\pi}{2}$ (b) $I_1=\dfrac{\pi}{2}I_2$
(c) $I_1+I_2=0$ (d) $I_1=I_2$

12. The area bounded by the curve $C : y = \tan x$, tangent drawn to C at $x=\dfrac{\pi}{4}$ and the x-axis is

(a) $\ln\sqrt{2}-\dfrac{1}{2}$ (b) $\ln\sqrt{2}-\dfrac{1}{4}$

(c) $\ln\sqrt{2}+\dfrac{1}{2}$ (d) $\ln\sqrt{2}+\dfrac{1}{4}$

13. If $y = x^y$, then the value of $\dfrac{x(1-y\log x)}{y^2}\cdot\dfrac{dy}{dx}$ is

(a) 0 (b) 1 (c) –1 (d) ± 1

14. The radius of a sphere is measured as 5 cm with an error possibly as large as 0.02 cm. The error and percentage error in computing the surface area of the sphere are

(a) 0.8π and 0.2% (b) 0.8π and 0.8%
(c) 0.4π and 0.4% (d) π and 1%

15. If $f(x) = \tan x - \tan^3 x + \tan^5 x - ....$ to ∞ with $0 < x < \pi/4$, then $\int_0^{\pi/4} f(x)\,dx =$

(a) 1 (b) 0 (c) 1/4 (d) 1/2

16. The larger area bounded by $y^2 = 4x$ and $x^2 + y^2 - 2x - 3 = 0$ is

(a) $2\pi - \frac{4}{3}$ (b) $2\pi + \frac{4}{3}$

(c) $2\pi + \frac{8}{3}$ (d) $2\pi + \frac{2}{3}$

17. The derivative of $a^{\sec x}$ w.r.t. $a^{\tan x}$ $(a > 0)$ is

(a) $\sec x\, a^{\sec x - \tan x}$ (b) $\sin x\, a^{\tan x - \sec x}$

(c) $\sin x\, a^{\sec x - \tan x}$ (d) $a^{\sec x - \tan x}$

18. Let V_1 be the volume of the largest cylinder which can be inscribed in a sphere of volume V_2, then V_2/V_1 equals

(a) $\sqrt{2}$ (b) $\sqrt{3}/2$

(c) $\sqrt{3}$ (d) $\sqrt{(3/2)}$

19. $\lim_{n\to\infty}\left(\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + ... + \frac{1}{2n}\right) =$

(a) ln 2 (b) ln 3 (c) ln(–2) (d) ln 4

20. The area bounded by the curves $y = \sqrt{x}, 2y + 3 = x$ and x-axis in the 1st quadrant is

(a) 9 (b) $\frac{27}{4}$ (c) 36 (d) 18

21. If $\sin(x + y) + \cos(x + y) = \log(x + y)$, then $\frac{d^2y}{dx^2} =$

(a) $-y/x$ (b) 0 (c) –1 (d) 1

22. The value of $\int_{-\pi}^{\pi} \frac{\sin^2 x}{1 + a^x}\,dx, \; a > 0$ is

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

23. $\int_{-3}^{2} |2x - 1|\,dx =$

(a) 10 (b) 6

(c) 15 (d) None of these

24. The area bounded by the curves $y = (x + 1)^2, y = (x - 1)^2$ and the line $y = \frac{1}{4}$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

25. If $f(x)$ is a function such that $f''(x) + f(x) = 0$ and $g(x) = [f(x)]^2 + [f'(x)]^2$ and $g(3) = 3$ then $g(8) =$

(a) 5 (b) 0 (c) 3 (d) 8

LEVEL - 2

26. If $f(x) = \min\{1, x^2, x^3\}$, then

(a) $f(x)$ is not continuous $\forall\, x \in R$

(b) $f(x) > 0, \forall\, x \in R$

(c) $f(x)$ is continuous but not differentiable $\forall\, x \in R$

(d) $f(x)$ is not differentiable at two points

27. Let $f: (0, \infty) \to R$ be given by

$$f(x) = \int_{1/x}^{x} e^{-\left(t + \frac{1}{t}\right)} \frac{dt}{t}.$$ Then

(a) $f(x)$ is monotonically decreasing on $[1, \infty)$

(b) $f(x)$ is monotonically decreasing on $(0, 1)$

(c) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$

(d) None of these

28. $\int \frac{dx}{e^{2x} - 3e^x} =$

(a) $\frac{1}{3e^x} - \frac{x}{9} + \frac{1}{9}\log(e^x + 3) + C$

(b) $\frac{1}{3e^x} - \frac{x}{9} + \frac{1}{9}\log(e^x - 3) + C$

(c) $-\frac{1}{3e^x} - \frac{x}{9} + C$

(d) $-\frac{1}{3e^x} - \frac{1}{9}\log(e^x + 3) + C$

29. The area of the region bounded by $x^2 + y^2 - 2y - 3 = 0$ and $y = |x| + 1$ is

(a) $\pi/2$ (b) π (c) 2π (d) 3π

30. The set of points where $f(x) = \frac{x}{1 + |x|}$ is differentiable, is

(a) $(-\infty, 0) \cup (0, \infty)$ (b) $(-\infty, -1) \cup (-1, \infty)$

(c) $(-\infty, \infty)$ (d) $(0, \infty)$

31. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is

(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/6$ (c) $\pi/4$

32. The value of the integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9 - x} + \sqrt{x}}\,dx$ is

(a) 1/2 (b) 3/2 (c) 2 (d) 1

33. The value of $\int_0^1 \frac{2^{x+1} - 5^{x-1}}{10^x}\,dx$ is

(a) $\frac{8}{5}\log_5 e - \frac{1}{10}\log_2 e$ (b) $\frac{8}{5}\log 5 - \frac{\log 2}{10}$

(c) $\frac{8}{5\log 5} + \frac{1}{10\log 2}$ (d) $\frac{1}{10\log 2} - \frac{8}{5\log 5}$

34. If $x^m \cdot y^n = (x + y)^{m+n}$, then dy/dx is

(a) $\frac{y}{x}$ (b) $\frac{x + y}{xy}$ (c) xy (d) $\frac{x}{y}$

35. If m and M are the minimum and the maximum values of $4+\frac{1}{2}\sin^2 2x-2\cos^4 x, x\in R$, then $M-m$ is equal to

(a) $\frac{9}{4}$ (b) $\frac{15}{4}$ (c) $\frac{7}{4}$ (d) $\frac{1}{4}$

36. $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx=$

(a) $\frac{\sqrt{2x^4-2x^2+1}}{x^2}+c$ (b) $\frac{\sqrt{2x^4-2x^2+1}}{x^3}+c$

(c) $\frac{\sqrt{2x^4-2x^2+1}}{x}+c$ (d) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2}+c$

37. The area (in square units) of the region bounded by the y-axis and the curve $2x = y^2 - 1$ is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) 2

38. If $F(x)=\left(f\left(\frac{x}{2}\right)\right)^2+\left(g\left(\frac{x}{2}\right)\right)^2$ where $f''(x) = -f(x)$ and $g(x) = f'(x)$ and given that $F(5) = 5$, then $F(10)$ is equal to

(a) 5 (b) 10 (c) 0 (d) 15

39. Let $f(x)=\begin{cases} e^x, & 0\le x\le 1\\ 2-e^{x-1}, & 1<x\le 2\\ x-e, & 2<x\le 3\end{cases}$

and $g(x)=\int_0^x f(t)dt, x\in[1,3]$ then $g(x)$ has

(a) local maxima at $x = 1 + \ln 2$ and local minima at $x = e$

(b) local maxima at $x = 1$ and local minima at $x = 1 + \ln 2$

(c) no local maxima (d) no local minima

40. $\int_\alpha^\beta \sqrt{\frac{x-\alpha}{\beta-x}}\,dx$

(a) $\frac{\pi^2}{2}(\beta-\alpha)$ (b) $\int_\alpha^\beta \sqrt{\frac{\beta-x}{x-\alpha}}\,dx$

(c) $\frac{\pi}{2}(\beta-\alpha)$ (d) $\int_\alpha^\beta \sqrt{\frac{\beta+x}{\alpha+x}}\,dx$

41. For $a \in R$ (the set of all real numbers), $a \ne -1$,

$$\lim_{n\to\infty}\frac{(1^a+2^a+....+n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+...+(na+n)]}=\frac{1}{60}.$$

Then $a =$

(a) $7, \frac{-17}{2}$ (b) $-7, \frac{17}{2}$

(c) $\frac{7}{2}, 17$ (d) None of these

42. $f(x)=\begin{cases} x^p\sin\frac{1}{x}, & x>0\\ 0\quad , & x=0\end{cases}$ is continuous at $x = 0$ but not differentiable then p lies

(a) $p\in[1,\infty)$ (b) $p\in(0, 1]$

(c) $p\in(-\infty, 0)$ (d) $p = 0$

43. The set of all local maxima for $y = \cos x$ is

(a) $\{n\pi : n\in Z\}$ (b) $\{2n\pi : n \in Z\}$

(c) $\left\{\frac{n\pi}{2} : n\in Z\right\}$ (d) $\left\{\frac{n\pi}{3} : n\in Z\right\}$

44. $\lim_{n\to\infty}\frac{(n!)^{1/n}}{n}$ is equal to

(a) e (b) $1/e$

(c) $e-1$ (d) None of these

45. The area enclosed between the parabola $y = x^2 - x + 2$ and the line $y = x + 2$ in sq. units equals

(a) 8/3 (b) 1/3 (c) 2/3 (d) 4/3

46. $\frac{d}{dx}\left\{\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)-\tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)\right\}=$

(a) $\frac{1}{\sqrt{1-x^2}}$ (b) $\frac{-1}{\sqrt{1-x^2}}$

(c) $\frac{1}{1+x^2}$ (d) $-\frac{1}{1-x^2}$

47. If $f(x)=\int_0^x e^{t^2}(t-2)(t-3)dt$ for all $x \in (0, \infty)$, then

(a) f has a local maximum at $x = 3$

(b) f is decreasing on (2, 3)

(c) f has a local minimum at $x = 2$

(d) None of these

48. If $\varphi(t)=\begin{cases}1, & \text{for } 0\le t<1\\ 0, & \text{otherwise}\end{cases}$ then

$$\int_{-3000}^{3000}\left(\sum_{r'=2014}^{2016}\varphi(t-r')\varphi(t-2016)\right)dt=$$

(a) 2 (b) 1

(c) 0 (d) does not exist

49. Area (in square units) enclosed by $y = 0$, $2x + y = 2$ and $x + y = 2$ is

(a) 1/2 (b) 1/4

(c) 1 (d) 2

50. If $f(x)$ and $g(x)$ are two functions with $g(x) = x - \frac{1}{x}$ and $fog(x) = x^3 - \frac{1}{x^3}$, then $f'(x) =$

(a) $3x^2 + 3$ (b) $x^2 - \frac{1}{x^2}$

(c) $1 + \frac{1}{x^2}$ (d) $3x^2 + \frac{3}{x^4}$