PRACTICE PAPER 5

Section II of CUET (UG) is Domain specific. In this section of Mathematics 40 questions to be attempted out of 50.

Time : 45 minutes

1. The value of $\sin^2\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$ is equal to

(a) $\frac{4}{5}$ (b) $\frac{16}{25}$ (c) $\frac{9}{25}$ (d) $\frac{5}{3}$

2. Let $A=\begin{pmatrix}\alpha & 0\\ 1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}1 & 0\\ 5 & 1\end{pmatrix}$ be two matrices where α is a real number. Then

(a) $A^2 = B$ for some α (b) $A^2 \neq B$ for any α

(c) $A^2 = -B$ for some α (d) $|A^2| \neq |B|$ for any α

3. A and B are two events such that $P(A) \neq 0$, $P(B|A)$ is

(i) A is a subset of B

(ii) $A \cap B = \phi$ are respectively

(a) 1, 1 (b) 0, 1 (c) 0, 0 (d) 1, 0

4. $f(x) = \begin{cases} 3x-8, & \text{if } x \leq 5 \\ 2k, & \text{if } x > 5 \end{cases}$ is continuous, find k.

(a) $\frac{3}{7}$ (b) $\frac{7}{2}$ (c) $\frac{2}{7}$ (d) $\frac{4}{7}$

5. The value of $\begin{vmatrix} 4 & 4 & 4 \\ (a+a^{-1})^2 & (b+b^{-1})^2 & (c+c^{-1})^2 \\ (a-a^{-1})^2 & (b-b^{-1})^2 & (c-c^{-1})^2 \end{vmatrix}$ is

(a) 0 (b) $4abc$

(c) $4(abc)^{-1}$ (d) $4[abc + (abc)^{-1}]$

6. The area of the region bounded by the curves $y = x^2$ and $y = \sqrt{x}$ is (in square units)

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{5}{6}$

7. The vector equation of the plane through the point (2, 1, –1) and parallel to the plane $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k}) = 0$ is

(a) $\vec{r}\cdot(\hat{i}+9\hat{j}+11\hat{k}) = 6$ (b) $\vec{r}\cdot(\hat{i}-9\hat{j}+11\hat{k}) = 4$

(c) $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k}) = 6$ (d) $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k}) = 4$

8. Let $f : R \to R$ be defined by $f(x) = \frac{1}{x} \forall x \in R$, then f is

(a) onto (b) not defined

(c) one-one (d) bijective

9. If the vectors $4\hat{i}+11\hat{j}+m\hat{k}$, $7\hat{i}+2\hat{j}+6\hat{k}$ and $\hat{i}+5\hat{j}+4\hat{k}$ are coplanar, then m is equal to

(a) 38 (b) 0

(c) 10 (d) –10

10. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 = 2$

(a) Touch each other

(b) Cut each other at right angle

(c) Cut at an angle $\frac{\pi}{3}$

(d) Cut at an angle $\frac{\pi}{4}$

11. The probability distribution of x is

x	0	1	2	3
$P(x)$	0.2	k	k	$2k$

Find the value of k.

(a) 0.3 (b) 0.1 (c) 0.2 (d) 0.4

12. $\int \frac{1+\log x}{(1+x\log x)^2}\,dx$ is equal to

(a) $\frac{1}{1+x\log|x|}+C$ (b) $\frac{1}{1+\log|x|}+C$

(c) $\frac{-1}{1+x\log|x|}+C$ (d) $\log\left|\frac{1}{1+\log|x|}\right|+C$

13. The order of the differential equation $y = C_1e^{C_2+x} + C_3e^{C_4+x}$ is

(a) 1 (b) 3 (c) 2 (d) 4

14. The vector equation of the straight line $\frac{x-2}{1} = \frac{y}{-3} = \frac{1-z}{2}$ is

(a) $\vec{r} = 2\hat{i}+\hat{k}+t(\hat{i}+3\hat{j}+2\hat{k})$

(b) $\vec{r} = 2\hat{i}-\hat{k}+t(\hat{i}-3\hat{j}-2\hat{k})$

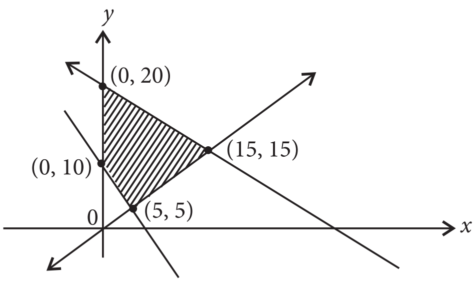
(c) $\vec{r} = 2\hat{i}+\hat{k}+t(\hat{i}-3\hat{j}+2\hat{k})$

(d) $\vec{r} = 2\hat{i}+\hat{k}+t(\hat{i}-3\hat{j}-2\hat{k})$

15. A relation R on {0, 1, 2} is given by R = {(0, 0), (1, 1), (0, 1), (2, 2), (1, 2)}. Then the relation R is

(a) reflexive
(b) symmetric
(c) transitive
(d) symmetric and transitive

16. The feasible region of an LPP is shown in the figure. If $z = 3x + 9y$, then the minimum value of z occurs at



(a) (5, 5) (b) (0, 10) (c) (0,20) (d) (15, 15)

17. At $x = 1$, the function $f(x) = \begin{cases} x^3 - 1, & 1 < x < \infty \\ x - 1, & -\infty < x \le 1 \end{cases}$ is

(a) continuous and differentiable
(b) continuous and non-differentiable
(c) discontinuous and differentiable
(d) discontinuous and non-differentiable

18. If the square of the matrix $\begin{bmatrix} a & b \\ a & -a \end{bmatrix}$ is the unit matrix, then b is equal to

(a) $\dfrac{a}{1+a^2}$ (b) $\dfrac{1-a^2}{a}$ (c) $\dfrac{1+a^2}{a}$ (d) $\dfrac{a}{1-a^2}$

19. $\int (1 - \tan^2 x)dx$ is equal to

(a) $\tan x + C$ (b) $\sec x + C$
(c) $2x - \sec x + C$ (d) $2x - \tan x + C$

20. A man speaks truth 2 out of 3 times. He picks one of the natural numbers in the set $S = \{1, 2, 3, 4, 5, 6, 7\}$ and reports that it is even. The probability that it is actually even is

(a) 2/5 (b) 1/10 (c) 1/5 (d) 3/5

21. Given $0 \le x \le \frac{1}{2}$ then the value of

$$\tan\left[\sin^{-1}\left\{\frac{x}{\sqrt{2}} + \frac{\sqrt{1-x^2}}{\sqrt{2}}\right\} - \sin^{-1} x\right] \text{ is}$$

(a) 1 (b) $\sqrt{3}$ (c) -1 (d) $\dfrac{1}{\sqrt{3}}$

22. The minimum value of the function $\max\{x, x^2\}$ is equal to

(a) 0 (b) 1 (c) 2 (d) $\dfrac{1}{2}$

23. $\displaystyle\int_0^{\sqrt{\pi}/2} 2x^3 \sin(x^2)dx =$

(a) $\dfrac{1}{\sqrt{2}}\left(1 + \dfrac{\pi}{4}\right)$ (b) $\dfrac{1}{\sqrt{2}}\left(1 - \dfrac{\pi}{4}\right)$

(c) $\dfrac{1}{\sqrt{2}}\left(\dfrac{\pi}{2} - 1\right)$ (d) $\dfrac{1}{\sqrt{2}}\left(1 - \dfrac{\pi}{2}\right)$

24. If $A = \{1, 3, 5, 7\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$, then the number of one-to-one functions from A into B is

(a) 1340 (b) 1860 (c) 1430 (d) 1680

25. If $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then the matrix A is

(a) $\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} -2 & 1 \\ 3 & -2 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$

26. The solution of $(2x - 10y^3)\dfrac{dy}{dx} + y = 0$ is

(a) $xy^2 = 2y^5 + C$ (b) $yx^2 = 2y^5 + C$
(c) $x^2y^2 = 2y^5 + C$ (d) None of these

27. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are coplanar and $|\vec{c}| = \sqrt{3}$, then

(a) $\alpha = \sqrt{2}, \beta = 1$ (b) $\alpha = 1, \beta = \pm 1$
(c) $\alpha = \pm 1, \beta = 1$ (d) $\alpha = \pm 1, \beta = -1$

28. If $\cos y = x\cos(a + y)$ with $\cos a \ne \pm 1$, then dy/dx is equal to

(a) $\dfrac{\sin a}{\cos^2(a+y)}$ (b) $\dfrac{\cos^2(a+y)}{\sin a}$

(c) $\dfrac{\cos a}{\sin^2(a+y)}$ (d) $\dfrac{\cos^2(a+y)}{\cos a}$

29. The angle between the lines whose direction cosines are $\left(\dfrac{\sqrt{3}}{4}, \dfrac{1}{4}, \dfrac{\sqrt{3}}{2}\right)$ and $\left(\dfrac{\sqrt{3}}{4}, \dfrac{1}{4}, \dfrac{-\sqrt{3}}{2}\right)$ is

(a) π (b) $\dfrac{\pi}{2}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{4}$

30. If $\vec{a} = (1, 2, 3)$, $\vec{b} = (2, -1, 1)$, $\vec{c} = (3, 2, 1)$ and $\vec{a} \times (\vec{b} \times \vec{c}) = \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}$, then

(a) $\alpha = 1, \beta = 10, \gamma = 3$ (b) $\alpha = 0, \beta = 10, \gamma = -3$
(c) $\alpha + \beta + \gamma = 8$ (d) $\alpha = \beta = \gamma = 0$

31. The value of $\tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right) + \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ is equal to

(a) $\tan^{-1}\left(\dfrac{5}{\sqrt{3}}\right)$ (b) $\tan^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$

(c) $\tan^{-1}\left(\frac{1}{2}\right)$ (d) $\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)$

32. The value of $\cos\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$ is

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{2}{5}$

33. If $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the inverse of the matrix $\begin{pmatrix} 1 & 5 \\ 7 & -3 \end{pmatrix}$, then d equals

(a) $\frac{-1}{38}$ (b) $\frac{-7}{38}$ (c) $\frac{3}{38}$ (d) $\frac{5}{38}$

34. The constant term in the expansion of

$$\begin{vmatrix} 3x+1 & 2x-1 & x+2 \\ 5x-1 & 3x+2 & x+1 \\ 7x-2 & 3x+1 & 4x-1 \end{vmatrix}$$ is

(a) 0 (b) – 10 (c) 2 (d) 6

35. Solution of $e^{\frac{dy}{dx}} = x$ when $x = 1$ and $y = 0$ is

(a) $y = x(\log x - 1) + 1$
(b) $y = x(\log x - 1) + 4$
(c) $y = x(\log x - 1) + 3$
(d) $y = x(\log x + 1) + 1$

36. If $f: R \to R$ is defined by $f(x) = 2x + 3$, then $f^{-1}(x)$

(a) does not exist because 'f' is not surjective
(b) is given by $\frac{x-3}{2}$ (c) is given by $\frac{1}{2x+3}$
(d) does not exist because 'f' is not injective

37. The rate of change of volume of a sphere with respect to its surface area when the radius is 4 cm is

(a) 2 cm^3/cm^2 (b) 4 cm^3/cm^2
(c) 8 cm^3/cm^2 (d) 6 cm^3/cm^2

38. The area bounded by $y = x + 2$, $y = 2 - x$ and the x-axis is (in square units)

(a) 1 (b) 2 (c) 4 (d) 6

39. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$, then $(AB)'$ is equal to

(a) $\begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & 7 \\ 10 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 7 \\ 10 & -2 \end{bmatrix}$

40. If $\vec{i} + \vec{j} - \vec{k}$ and $2\vec{i} - 3\vec{j} + \vec{k}$ are adjacent sides of a parallelogram, then the lengths of its diagonals are

(a) $\sqrt{21}, \sqrt{13}$ (b) $\sqrt{3}, \sqrt{14}$
(c) $\sqrt{13}, \sqrt{14}$ (d) $\sqrt{21}, \sqrt{3}$

41. The values of k for which the system
$(k + 1)x + 8y = 0$; $kx + (k + 3)y = 0$
has unique solution, are

(a) 3, 1 (b) –3, 1 (c) 3, –1 (d) –3, –1

42. If $f(x) = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, then the value of $(1 - x^2) f'(x) - xf(x)$ is

(a) 0 (b) 1 (c) 2 (d) 3

43. $\int_0^{\pi/2} \frac{\sin 2t}{\sin^4 t + \cos^4 t} dt =$

(a) π (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/2$

44. Let the probability distribution of a random variable X be given by

X	–1	0	1	2	3
$P(X)$	a	$2a$	$3a$	$4a$	$5a$

Then the expectation of X is

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{5}{3}$

45. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then x^2 is equal to

(a) $1 - y^2$ (b) y^2 (c) 0 (d) $\sqrt{1-y}$

46. The function $f(x) = [x]$ where $[x]$ is the greatest integer function is continuous at

(a) 1.5 (b) 4 (c) 1 (d) –2

47. The distance between the planes

$\vec{r} \cdot (\hat{i} + 2\hat{j} - 2\hat{k}) + 5 = 0$ and

$\vec{r} \cdot (2\hat{i} + 4\hat{j} - 4\hat{k}) - 16 = 0$ is

(a) 3 (b) $\frac{11}{3}$ (c) 13 (d) $\frac{13}{3}$

48. The function f given by $f(x) = (x^2 - 3)e^x$ is decreasing on the interval

(a) $(-3, \infty)$ (b) $(1, \infty)$ (c) $(-\infty, 1)$ (d) $(-3, 1)$

49. An integrating factor of the differential equation $xdy - ydx + x^2e^x dx = 0$ is

(a) $\frac{1}{x}$ (b) $\log\sqrt{1+x^2}$
(c) $\sqrt{1+x^2}$ (d) x

50. If A and B are matrices of order 3 and $|A| = 5$, $|B| = 3$, then $|3AB|$ is

(a) 425 (b) 405 (c) 565 (d) 585

ANSWER KEYS

1.	(b)	**2.**	(b)	**3.**	(d)	**4.**	(b)	**5.**	(a)	**6.**	(b)	**7.**	(c)	**8.**	(b)	**9.**	(c)	**10.**	(b)
11.	(c)	**12.**	(c)	**13.**	(a)	**14.**	(d)	**15.**	(a)	**16.**	(a)	**17.**	(b)	**18.**	(b)	**19.**	(d)	**20.**	(d)
21.	(a)	**22.**	(a)	**23.**	(b)	**24.**	(d)	**25.**	(a)	**26.**	(a)	**27.**	(c)	**28.**	(b)	**29.**	(c)	**30.**	(b)
31.	(a)	**32.**	(a)	**33.**	(a)	**34.**	(d)	**35.**	(a)	**36.**	(b)	**37.**	(a)	**38.**	(c)	**39.**	(b)	**40.**	(a)
41.	(d)	**42.**	(b)	**43.**	(d)	**44.**	(d)	**45.**	(a)	**46.**	(a)	**47.**	(d)	**48.**	(d)	**49.**	(a)	**50.**	(b)

Hints & Explanations

1. (b): We have, $\sin^2\left(\cos^{-1}\left(\frac{3}{5}\right)\right)=\left[\sin\left(\sin^{-1}\left(\frac{4}{5}\right)\right)\right]^2$

$\left[\because \cos^{-1}x=\sin^{-1}\sqrt{1-x^2}\right]$

$=\left(\frac{4}{5}\right)^2=\frac{16}{25}$

2. (b)

3. (d): (i) It is given that, $A\subset B$

$\Rightarrow\ A\cap B=A$

$\therefore\ P(B|A)=\frac{P(A\cap B)}{P(A)}=\frac{P(A)}{P(A)}=1$

(ii) If $A\cap B=\phi$

Then $P(B|A)=\frac{P(A\cap B)}{P(A)}=0$

4. (b): Since $f(x)$ is continuous

$\therefore\ \lim_{x\to 5^-} f(x)=\lim_{x\to 5^+} f(x)\Rightarrow \lim_{x\to 5}(3x-8)=2k$

$\Rightarrow\ 3(5)-8=2k\ \Rightarrow\ 2k=7\ \Rightarrow\ k=\frac{7}{2}$.

5. (a): Let $\Delta=\begin{vmatrix}4 & 4 & 4\\ (a+a^{-1})^2 & (b+b^{-1})^2 & (c+c^{-1})^2\\ (a-a^{-1})^2 & (b-b^{-1})^2 & (c-c^{-1})^2\end{vmatrix}$

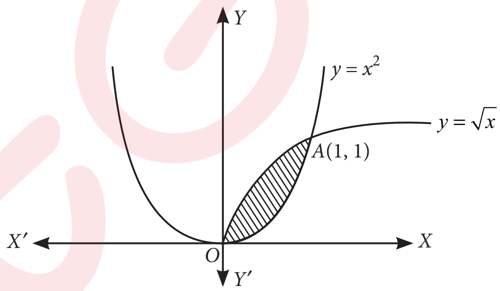
Using operation $R_2\to R_2-R_3$, we get

$$\Delta=\begin{vmatrix}4 & 4 & 4\\ 4aa^{-1} & 4bb^{-1} & 4cc^{-1}\\ (a-a^{-1})^2 & (b-b^{-1})^2 & (c-c^{-1})^2\end{vmatrix}$$

$$\Rightarrow\ \Delta=\begin{vmatrix}4 & 4 & 4\\ 4 & 4 & 4\\ (a-a^{-1})^2 & (b-b^{-1})^2 & (c-c^{-1})^2\end{vmatrix}=0$$

$[\because R_1$ and R_2 are identical rows$]$

6. (b): Required area $=\int_0^1(\sqrt{x}-x^2)dx$



$=\left[\frac{2}{3}x^{3/2}-\frac{x^3}{3}\right]_0^1$

$=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$ sq. units

7. (c) : The equation of a plane parallel to the plane $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=0$ is $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=d$ ...(i)

Since it passes through (2, 1, –1)

$\therefore\ (2\hat{i}+\hat{j}-\hat{k})\cdot(\hat{i}+3\hat{j}-\hat{k})=d$

$\Rightarrow\ 2+3+1=d\Rightarrow\ d=6$

Putting $d=6$ in (i), we get $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=6$

8. (b): Given function is $f(x)=\frac{1}{x}$

Since $f: R\to R$, but for $x=0$, function is not defined

Also, we do not get any image for $x=0$

$\therefore$ f is not defined in the given interval.

9. (c) : Since, $4\hat{i}+11\hat{j}+m\hat{k}, 7\hat{i}+2\hat{j}+6\hat{k}$ and $\hat{i}+5\hat{j}+4\hat{k}$ are coplanar

$$\therefore\ \begin{vmatrix}4 & 11 & m\\ 7 & 2 & 6\\ 1 & 5 & 4\end{vmatrix}=0$$

$\Rightarrow \quad 4(8-30) - 11(28-6) + m(35-2) = 0$

$\Rightarrow \quad -88 - 242 + 33m = 0 \Rightarrow m = 10$

10. (b) : Given curves are $x^3 - 3xy^2 + 2 = 0$ ...(i)

and $3x^2y - y^3 = 2$ ...(ii)

Differentiating (i) w.r.t. x, we get

$$3x^2 - 3\left[x(2y)\frac{dy}{dx} + y^2\right] = 0 \Rightarrow x^2 - 2xy\frac{dy}{dx} - y^2 = 0$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{x^2 - y^2}{2xy} \quad \text{...(iii)}$$

Differentiating (ii) w.r.t. x, we get

$$3\left(x^2\frac{dy}{dx} + y(2x)\right) - 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow (x^2 - y^2)\frac{dy}{dx} = -2xy$$

$$\Rightarrow m_2 = \frac{dy}{dx} = -\frac{2xy}{x^2 - y^2} \quad \text{...(iv)}$$

$\because \quad m_1 m_2 = -1$ [From (iii) & (iv)]

Hence, given curves cut each other at right angle.

11. (c) : Since, sum of all probabilities = $\Sigma P(x) = 1$

$$\Rightarrow 0.2 + k + k + 2k = 1 \Rightarrow 4k = 0.8 \Rightarrow k = \frac{0.8}{4} = 0.2$$

12. (c) : Let $I = \int \frac{1 + \log x}{(1 + x\log x)^2}dx$

Put $x\log x = t \Rightarrow (1 + \log x)dx = dt$

$$\Rightarrow I = \int \frac{dt}{(1+t)^2} = \frac{-1}{1+t} + C = \frac{-1}{1 + x\log|x|} + C$$

13. (a)

14. (d) : We have, $\frac{x-2}{1} = \frac{y}{-3} = \frac{z-1}{-2} = t$ (say)

$\Rightarrow \quad x = t + 2,\ y = -3t,\ z = -2t + 1$

So, the vector equation of the line is,

$$x\hat{i} + y\hat{j} + z\hat{k} = (t+2)\hat{i} - 3t\hat{j} + (-2t+1)\hat{k}$$

$$\Rightarrow \vec{r} = (2\hat{i} + \hat{k}) + t(\hat{i} - 3\hat{j} - 2\hat{k})$$

15. (a) : We have, $R = \{(0, 0), (1, 1), (0, 1), (2, 2), (1, 2)\}$

It is clear that, R is reflexive as $(0, 0), (1, 1), (2, 2) \in R$

Now, as $(1, 2) \in R$ but $(2, 1) \notin R$

So, R is not symmetric.

Also, $(0, 1) \in R$, $(1, 2) \in R$ but $(0, 2) \notin R$

So, R is not transitive.

16. (a)

17. (b) : $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1}(x^3 - 1) = 0$

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1}(x - 1) = 0$

Also, $f(1) = 0 \Rightarrow f$ is continuous

$$f'(x) = \begin{cases} 3x^2, & 1 < x < \infty \\ 1, & -\infty < x < 1 \end{cases}$$

$f'(1^+) = 3,\ f'(1^-) = 1$

$\Rightarrow f$ is not differentiable

18. (b) : According to question,

$$\begin{bmatrix} a & b \\ a & -a \end{bmatrix}\begin{bmatrix} a & b \\ a & -a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2 + ab & 0 \\ 0 & ab + a^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing, we get $a^2 + ab = 1 \quad \Rightarrow b = \frac{1 - a^2}{a}$

19. (d) : Let $I = \int (1 - \tan^2 x)dx = \int (2 - (1 + \tan^2 x))dx$

$$= 2x - \int \sec^2 x\,dx = 2x - \tan x + C$$

20. (d)

21. (a) : Let, $y = \tan\left[\sin^{-1}\left\{\frac{x}{\sqrt{2}} + \frac{\sqrt{1-x^2}}{\sqrt{2}}\right\} - \sin^{-1}x\right]$

Put $x = \sin\theta$, we get

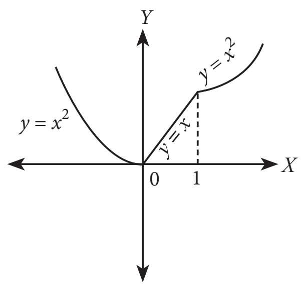
$$y = \tan\left[\sin^{-1}\left[\frac{\sin\theta}{\sqrt{2}} + \frac{\sqrt{1 - \sin^2\theta}}{\sqrt{2}}\right] - \sin^{-1}(\sin\theta)\right]$$

$$= \tan\left[\sin^{-1}\left[\frac{\sin\theta + \cos\theta}{\sqrt{2}}\right] - \theta\right]$$

$$= \tan\left[\sin^{-1}\left\{\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right\} - \theta\right]$$

$$= \tan\left[\sin^{-1}\left\{\sin\left(\theta + \frac{\pi}{4}\right)\right\} - \theta\right] = \tan\frac{\pi}{4} = 1$$

22. (a) : Graph of $\max\{x, x^2\}$ is shown below



Hence, min value of $\max\{x, x^2\}$ is 0.

23. (b) : $\int_0^{\sqrt{\pi}/2} 2x^3 \sin x^2\,dx = \int_0^{\pi/4} x\sin x\,dx$

(By Putting $x^2 = t$)

$= [x(-\cos x) - (-\sin x)]_0^{\pi/4}$ (Using Integration by parts)

$$= -\frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(1 - \frac{\pi}{4}\right)$$

24. (d): Number of one-one functions from A to B
$= {}^nC_m \times m!$ where $n \geq m$ $= {}^8C_4 \times 4! = 1680$

25. (a): We have, $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $BA = I$ (say) $\Rightarrow A = B^{-1}$

Now, $\text{adj } B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}' = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

and $|B| = 4 - 3 = 1$

$\therefore$ $B^{-1} = \frac{(\text{adj } B)}{|B|} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

So, $A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

26. (a): Given differential equation is

$(2x - 10y^3)\frac{dy}{dx} + y = 0$ or $\frac{dy}{dx} = \frac{-y}{2x - 10y^3}$

or $\frac{dx}{dy} = \frac{2x - 10y^3}{-y} = \frac{-2x}{y} + 10y^2$ or $\frac{dx}{dy} + \frac{2}{y}x = 10y^2$

$\Rightarrow$ I.F. $= e^{\int \frac{2}{y}dy} = e^{2\log y} = y^2$

$\therefore$ Required solution is $x \cdot y^2 = \int 10y^2 \cdot y^2\, dy + C$

or $x \cdot y^2 = 10 \cdot \frac{y^5}{5} + C$ or $xy^2 = 2y^5 + C$

27. (c): Since given vectors are coplanar

$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0 \Rightarrow \beta = 1$ Since $|\vec{c}| = \sqrt{3}$

$\Rightarrow \sqrt{1 + \alpha^2 + \beta^2} = \sqrt{3} \Rightarrow \alpha^2 + \beta^2 = 2 \Rightarrow \alpha^2 = 1$

$\therefore$ $\alpha = \pm 1$

28. (b)

29. (c): Let θ be the angle between lines.

$$\therefore \cos\theta = \left|\frac{\sqrt{3}}{4} \times \frac{\sqrt{3}}{4} + \frac{1}{4} \times \frac{1}{4} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right| = \left|\frac{-1}{2}\right| = \frac{1}{2}$$

$\Rightarrow \theta = \frac{\pi}{3}$

30. (b): $\vec{a} \times (\vec{b} \times \vec{c}) = \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}$

$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}$

Comparing like coefficients, we get

$\alpha = 0, \beta = \vec{a} \cdot \vec{c} = 3 + 4 + 3 = 10, \gamma = -(\vec{a} \cdot \vec{b}) = -(2 - 2 + 3) = -3$

31. (a): $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}}{1 - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}}\right)$

$= \tan^{-1}\left(\frac{3 + 2}{2\sqrt{3} - \sqrt{3}}\right) = \tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$

32. (a): Let $\tan^{-1}\left(\frac{3}{4}\right) = \theta$

$\Rightarrow$ $\tan\theta = \frac{3}{4}$

$\therefore$ $\cos\theta = \frac{4}{5}$

Now, $\cos\left(\tan^{-1}\left(\frac{3}{4}\right)\right) = \cos\theta = \frac{4}{5}$

33. (a): Let $A = \begin{bmatrix} 1 & 5 \\ 7 & -3 \end{bmatrix}$ $\therefore A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ [Given]

Now $AA^{-1} = I \Rightarrow \begin{bmatrix} 1 & 5 \\ 7 & -3 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a + 5c & b + 5d \\ 7a - 3c & 7b - 3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now $b + 5d = 0$ ...(i) and $7b - 3d = 1$ ...(ii)

Solving (i) and (ii), we get $d = \frac{-1}{38}$.

34. (d)

35. (a): $e^{\frac{dy}{dx}} = x \Rightarrow \frac{dy}{dx} = \log x \Rightarrow \int dy = \int(\log x)dx$

$\Rightarrow$ $y = x \cdot \log x - x + C$

Putting $x = 1, y = 0$ we get $C = 1$

$\Rightarrow$ $y = x\log x - x + 1$

36. (b): $f(x) = 2x + 3 \Rightarrow f^{-1}(x) = \frac{x - 3}{2}$

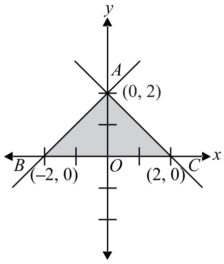
37. (a): $\frac{dV}{dS} = \frac{dV/dt}{dS/dt} = \frac{4\pi r^2 \frac{dr}{dt}}{8\pi r \frac{dr}{dt}} = \frac{r}{2} = \frac{4}{2} = 2\text{ cm}^3/\text{cm}^2$

38. (c): $y = x + 2$ ...(i)

$y = 2 - x$ ...(ii)

Point of intersection of two lines are (0, 2)

Area bounded by lines



$$= \int_{-2}^{0} (x+2)dx + \int_0^2 (2-x)dx$$

$$= \left[\frac{x^2}{2} + 2x\right]_{-2}^{0} + \left[2x - \frac{x^2}{2}\right]_0^2$$

$$= 0 - \left(\frac{4}{2} - 4\right) + \left(4 - \frac{4}{2}\right) - 0$$

$$= \left(\frac{4}{2}\right) + \left(\frac{4}{2}\right) = 4 \text{ square units}$$

39. (b): $AB = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2-6+1 & 1-4+1 \\ 4+3+3 & 2+2+3 \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$$

$$\Rightarrow (AB)' = \begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$$

40. (a): $|\vec{a} + \vec{b}| = \sqrt{13}$ and $|\vec{a} - \vec{b}| = \sqrt{21}$

41. (d): The given system of equations is

$(k + 1)x + 8y = 0,\ kx + (k + 3)y = 0$

Coefficient matrix, $A = \begin{bmatrix} k+1 & 8 \\ k & k+3 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} k+1 & 8 \\ k & k+3 \end{vmatrix} = (k + 1)(k + 3) - 8k$

$= k^2 + 4k + 3 - 8k = k^2 - 4k + 3 = (k - 1)(k - 3)$

For unique solution $|A| \neq 0$ *i.e.*, k must not be equal to 1 or 3.

42. (b): We have, $f(x) = \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$

Differentiating w.r.t. x, we get

$$f'(x) = \frac{\sqrt{1-x^2} \times \frac{1}{\sqrt{1-x^2}} - \sin^{-1} x \times \frac{1}{2\sqrt{1-x^2}} \times (-2x)}{1-x^2}$$

$\Rightarrow\ (1 - x^2) f'(x) - x f(x) = 1$

43. (d)

44. (d): As we know that, $\Sigma P(X) = 1$

$\Rightarrow\ a + 2a + 3a + 4a + 5a = 1 \Rightarrow\ 15a = 1\ \Rightarrow\ a = \dfrac{1}{15}$

So, probability distribution becomes,

X	–1	0	1	2	3
$P(X)$	1/15	2/15	3/15	4/15	5/15

$\therefore\ E(X) = \Sigma X \cdot P(X)$

$$= (-1)\left(\frac{1}{15}\right) + 0 + 1\left(\frac{3}{15}\right) + 2\left(\frac{4}{15}\right) + 3\left(\frac{5}{15}\right)$$

$$= \frac{-1+3+8+15}{15} = \frac{25}{15} = \frac{5}{3}$$

45. (a): Given $\sin^{-1} x + \sin^{-1} y = \dfrac{\pi}{2}$

Let $\sin^{-1} x = t \Rightarrow\ x = \sin t$

$\Rightarrow\ t + \sin^{-1} y = \dfrac{\pi}{2} \Rightarrow \sin^{-1} y = \dfrac{\pi}{2} - t$

$\Rightarrow\ y = \sin\left(\dfrac{\pi}{2} - t\right) = \cos t$

$\therefore\ x^2 = \sin^2 t = 1 - \cos^2 t = 1 - y^2$

46. (a)

47. (d): The equation of two planes is

$x + 2y - 2z + 5 = 0$ and $2x + 4y - 4z - 16 = 0$

Distance between the planes $= \dfrac{5-(-8)}{\sqrt{1^2+2^2+2^2}} = \dfrac{13}{3}$

48. (d): We have, $f(x) = (x^2 - 3)e^x$

Differentiating both sides, we get

$f'(x) = (x^2 - 3)\cdot e^x + e^x \cdot 2x = e^x(x^2 + 2x - 3)$

$= e^x (x + 3)(x - 1)$

For $f(x)$ to be decreasing, we should have $f'(x) \leq 0$

$\Rightarrow\ e^x (x + 3)(x - 1) \leq 0$

Sign of $f'(x)$: $-\infty$ to -3: +ve; -3 to 1: –ve; 1 to ∞: +ve

$\therefore\ f(x)$ is decreasing in the interval $(-3, 1)$.

49. (a): $xdy - ydx + x^2e^x dx = 0$ …(i)

Dividing (i) by dx, we get

$$x\frac{dy}{dx} - y + x^2e^x = 0 \Rightarrow \frac{dy}{dx} - \frac{y}{x} + xe^x = 0 \Rightarrow \frac{dy}{dx} - \frac{y}{x} = -xe^x$$

It is the differential equation of the form

$\dfrac{dy}{dx} + P\cdot y = Q(x)$, where $P = -\dfrac{1}{x}$, $Q = -xe^x$

$$\text{I.F.} = e^{\int P dx} = e^{\int \frac{-1}{x} dx} = e^{[\log x^{-1}]} = x^{-1} = \frac{1}{x}$$

50. (b): $|3AB| = 3^3\,|AB| = 3^3 \cdot |A| \cdot |B|$

$= 27 \times 5 \times 3 = 405$

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