PRACTICE PAPER

MATHEMATICS

51. If $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$ and $\vec{c} = \hat{k} + \hat{i}$, a unit vector parallel to $\vec{a} + \vec{b} + \vec{c}$ is

(a) $2\hat{i} + 2\hat{j} + 2\hat{k}$ (b) $\dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

(c) $\dfrac{\hat{i} + \hat{j} + \hat{k}}{2\sqrt{3}}$ (d) $\dfrac{\vec{a} + \vec{b} + \vec{c}}{\sqrt{3}}$

52. $\displaystyle\int \frac{x^2 + 1}{x^4 + 1}\, dx =$

(a) $\dfrac{1}{\sqrt{2}} \tan^{-1} (x^2 + 1) + c$

(b) $\dfrac{1}{\sqrt{2}} \tan^{-1} \dfrac{x^2 + 1}{\sqrt{2}x} + c$

(c) $\dfrac{1}{\sqrt{2}} \tan^{-1} (x^2 - 1) + c$

(d) $\dfrac{1}{\sqrt{2}} \tan^{-1} \dfrac{x^2 - 1}{\sqrt{2}x} + c$

53. If $x = a(\cos\theta + \theta \sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, $\dfrac{dy}{dx} =$

(a) $\cot\theta$ (b) $\tan\theta$

(c) $\tan\dfrac{\theta}{2}$ (d) $\cot\dfrac{\theta}{2}$

54. If $aN = \{an : n \in N\}$ and $bN \cap cN = dN$, where $a, b, c \in N$ and b, c are co-prime, then

(a) $b = cd$ (b) $c = bd$

(c) $d = bc$ (d) none of these

55. The equation of straight line through the intersection of the lines $x - 2y = 1$ and $x + 3y = 2$ and parallel to $3x + 4y = 0$, is

(a) $3x + 4y + 5 = 0$ (b) $3x + 4y - 10 = 0$

(c) $3x + 4y - 5 = 0$ (d) $3x + 4y + 6 = 0$

56. The area bounded by $y = \tan^{-1}x$, $x = 1$ and X-axis is

(a) $\left(\dfrac{\pi}{4} + \log\sqrt{2}\right)$ sq. unit

(b) $\left(\dfrac{\pi}{4} - \log\sqrt{2}\right)$ sq. unit

(c) $\left(\dfrac{\pi}{4} - \log\sqrt{2} + 1\right)$ sq. units

(d) none of these

57. The general solution of the differential equation $\dfrac{dy}{dx} + \sin\left(\dfrac{x+y}{2}\right) = \sin\left(\dfrac{x-y}{2}\right)$ is

(a) $\log\tan\left(\dfrac{y}{2}\right) = C - 2\sin x$

(b) $\log\tan\left(\dfrac{y}{4}\right) = C - 2\sin\left(\dfrac{x}{2}\right)$

(c) $\log\tan\left(\dfrac{y}{2} + \dfrac{\pi}{4}\right) = C - 2\sin x$

(d) $\log\tan\left(\dfrac{y}{4} + \dfrac{\pi}{4}\right) = C - 2\sin\left(\dfrac{x}{2}\right)$

58. If $f(x) = xe^{x(1-x)}$, then $f(x)$ is

(a) increasing on $\left[-\dfrac{1}{2}, 1\right]$

(b) decreasing on R

(c) increasing on R

(d) decreasing on $\left[-\dfrac{1}{2}, 1\right]$

59. A man throws a fair coin a number of times and gets 2 points for each head he throws and 1 point for each tail he throws. The probability that he gets exactly 6 points is

(a) $\dfrac{21}{32}$ (b) $\dfrac{23}{32}$ (c) $\dfrac{41}{64}$ (d) $\dfrac{43}{64}$

60. $\left| \dfrac{1}{(2+i)^2} - \dfrac{1}{(2-i)^2} \right| =$

(a) $\dfrac{\sqrt{8}}{5}$ (b) $\dfrac{25}{8}$ (c) $\dfrac{5}{\sqrt{8}}$ (d) $\dfrac{8}{25}$

61. The equation of the circle described on the line joining the points $(-2, -1)$ and $(3, 4)$ as diameter is

(a) $x^2 + y^2 + x + 3y + 10 = 0$

(b) $x^2 + y^2 - x + 3y + 10 = 0$

(c) $x^2 + y^2 - x - 3y - 10 = 0$

(d) $x^2 + y^2 + x + 3y - 10 = 0$

62. The number of integer solutions for the equation $x + y + z + t = 20$ where x, y, z, t are all ≥ -1 is

(a) ${}^{20}C_4$ (b) ${}^{23}C_3$

(c) ${}^{27}C_4$ (d) ${}^{27}C_3$

63. If $n > 1$ then $(1 + x)^n - 1 - nx$ is divisible by

(a) x^2 (b) x^5 (c) x^3 (d) x^4

64. If $A = \begin{bmatrix} 0 & 3 \\ -7 & 5 \end{bmatrix}$, and $kA^2 = 5A - 21I$, then $k =$

(a) 1 (b) 2 (c) 3 (d) 4

65. If the foot of perpendicular from the point $(1, -5, -10)$ to the plane $x - y + z = 5$ is the point (a, b, c), then $a + b + c$ is

(a) 10 (b) – 10

(c) 11 (d) – 11

66. Let $f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & x<0 \\ c, & x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}}, & x>0 \end{cases}$

If $f(x)$ is continuous at $x = 0$, then

(a) $a + c = 0, b = 1$ (b) $a + c = 1, b \in R$
(c) $a + c = -1, b \in R$ (d) $a + c = -1, b = -1$

67. If the function f satisfies the relation $f(x + y) = f(x) \cdot f(y)$ for all natural numbers x, y, $f(1) = 2$ and $\sum_{r=1}^{n} f(a+r) = 16(2^n - 1)$, then the natural number a is

(a) 2 (b) 3 (c) 4 (d) 5

68. The successive terms of an A.P. are a_1, a_2, a_3, ... If $a_6 + a_9 + a_{12} + a_{15} = 20$, then $\sum_{r=1}^{20} a_r =$

(a) 75 (b) 100 (c) 120 (d) 150

69. The contrapositive of "If $x \in A \cap B$, then $x \in A$ and $x \in B$" is

(a) If $x \notin A$ and $x \notin B$, then $x \notin A \cap B$
(b) If $x \notin A$ or $x \notin B$, then $x \notin A \cap B$
(c) If $x \notin A$ or $x \notin B$, then $x \notin A \cup B$
(d) If $x \notin A$ or $x \notin B$, then $x \in A \cap B$

70. A ray of light along the line $x = 3$ is reflected at the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. The slope of the reflected ray is

(a) $\frac{4}{15}$ (b) $\frac{8}{15}$ (c) $\frac{15}{8}$ (d) $\frac{15}{4}$

Numerical Value Type

71. $\lim_{x \to 0} \left[\frac{x}{\sqrt{1+x} - \sqrt{1-x}} \right] =$ _________.

72. The number of solutions of the equation $1 + \sin x \sin^2 \frac{x}{2} = 0$ in $[-\pi, \pi]$ is _________.

73. A is a matrix of order 3 and $|A| = 8$. Then $|\text{adj } A| =$ _________.

74. The integer k for which the inequality $x^2 - 2(4k - 1)x + 15k^2 - 2k - 7 > 0$ is valid for any real x, is _________.

75. The A.M. and S.D. of 100 items was recorded as 40 and 5.1 respectively. Later on it was discovered that one observation 40 was wrongly copied down as 50. Then the correct S.D is _________.

HINTS & EXPLANATIONS

MATHEMATICS

51. (b) : $\vec{a}+\vec{b}+\vec{c} = 2\hat{i}+2\hat{j}+2\hat{k} = 2(\hat{i}+\hat{j}+\hat{k})$

$\therefore$ Unit vector parallel to $\vec{a}+\vec{b}+\vec{c}$

$$= \frac{2(\hat{i}+\hat{j}+\hat{k})}{\sqrt{12}} = \frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$$

52. (d) : $\int \frac{x^2+1}{x^4+1}\,dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx$

$$= \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}\,dx$$

Put $x-\frac{1}{x}=t$, so that $\left(1+\frac{1}{x^2}\right)dx = dt$

$$\therefore \int \frac{x^2+1}{x^4+1}\,dx = \int \frac{dt}{t^2+2} = \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}}+c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$$

53. (b) : $x = a(\cos\theta+\theta\sin\theta)$, $y = a(\sin\theta-\theta\cos\theta)$

$$\frac{dx}{d\theta} = a(-\sin\theta+\theta\cos\theta+\sin\theta) = a\theta\cos\theta$$

$$\frac{dy}{d\theta} = a(\cos\theta+\theta\sin\theta-\cos\theta) = a\theta\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$

54. (c) : Given, $aN = \{an : n \in N\}$

$\therefore$ $bN = \{bn : n \in N\}$ and $cN = \{cn : n \in N\}$

Also, given $bN \cap cN = dN$

$\therefore$ $bc \in bN \cap cN$ or $bc \in dN$

$\therefore$ $bc = d$ $\quad$ ($\because$ b and c are co-prime)

55. (c) : The intersection point of lines $x - 2y = 1$ and $x + 3y = 2$ is $\left(\frac{7}{5},\frac{1}{5}\right)$.

Required line is parallel to $3x + 4y = 0$.

$\therefore$ The slope of required line $= -\frac{3}{4}$

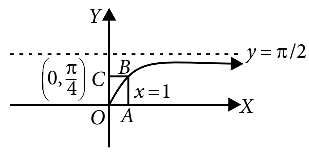
$\therefore$ Equation of required line which passes through $\left(\frac{7}{5},\frac{1}{5}\right)$ and having slope, $-\frac{3}{4}$ is

$$y-\frac{1}{5} = \frac{-3}{4}\left(x-\frac{7}{5}\right) \Rightarrow \frac{3x}{4}+y = \frac{21}{20}+\frac{1}{5}$$

$$\Rightarrow \frac{3x+4y}{4} = \frac{21+4}{20} \Rightarrow 3x+4y=5$$

$$\Rightarrow 3x+4y-5=0$$

56. (b) : Required area = Area of rectangle $OABC$ – Area of curve $OBCO$



$$= \frac{\pi}{4} - \int_0^{\pi/4}\tan y\,dy = \frac{\pi}{4}+[\log\cos y]_0^{\pi/4}$$

$$= \frac{\pi}{4}+\log\cos\frac{\pi}{4}-\log\cos(0)$$

$$= \frac{\pi}{4}+\log 1-\log\sqrt{2}-\log 1 = \frac{\pi}{4}-\log\sqrt{2}$$

57. (b) : Given equation is

$$\frac{dy}{dx}+\sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \sin\left(\frac{x-y}{2}\right)-\sin\left(\frac{x+y}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = -2\sin\left(\frac{y}{2}\right)\cos\left(\frac{x}{2}\right)$$

$$\Rightarrow \operatorname{cosec}\left(\frac{y}{2}\right)dy = -2\cos\left(\frac{x}{2}\right)dx$$

On integrating both sides, we get

$$\int \operatorname{cosec}\left(\frac{y}{2}\right)dy = -\int 2\cos\left(\frac{x}{2}\right)dx + C_1$$

$$\Rightarrow \frac{\log\left(\tan\frac{y}{4}\right)}{\frac{1}{2}} = -\frac{2\sin\left(\frac{x}{2}\right)}{\frac{1}{2}}+C_1$$

$$\Rightarrow \log\left(\tan\frac{y}{4}\right) = C-2\sin\left(\frac{x}{2}\right)$$

58. (a) : $f(x) = xe^{x(1-x)}$

On differentiating w.r.t. x, we get

$f'(x) = e^{x(1-x)} + x \times e^{x(1-x)} \times (1-2x)$

$= e^{x(1-x)} \times \{1 + x(1-2x)\}$

$= e^{x(1-x)} \times (-2x^2 + x + 1)$

It is clear that $e^{x(1-x)} > 0$ for all x.

Now, by sign rule for $-2x^2 + x + 1$, we have

$f'(x) \geq 0$, if $x \in \left[-\frac{1}{2},1\right]$

so, $f(x)$ is increasing on $\left[-\frac{1}{2},1\right]$.

59. (d) : Required probability

$= P(\text{HHH}) + P(\text{HHTT}) + P(\text{HTTTT}) + P(\text{TTTTTT})$

$= \frac{1}{2^3} + \left({}^4C_2\right)\frac{1}{2^4} + \left({}^5C_1\right)\frac{1}{2^5} + \frac{1}{2^6} = \frac{43}{64}$

60. (d) : $\left|\frac{1}{(2+i)^2} - \frac{1}{(2-i)^2}\right|$

$= \left|\frac{(2-i)^2 - (2+i)^2}{(2+i)^2\ (2-i)^2}\right| = \left|\frac{-8i}{25}\right| = \frac{8}{25}.$

61. (c) : The equation of the circle is

$(x + 2)(x - 3) + (y + 1)(y - 4) = 0$

$\Rightarrow\ x^2 + y^2 - x - 3y - 10 = 0.$

62. (d) : Let $u = x + 1$, $v = y + 1$, $w = z + 1$ and $p = t + 1$

$\therefore\ u, v, w, p \geq 0$ and $u + v + w + p = 24$.

So, required number of solutions is

${}^{24+4-1}C_{4-1} = {}^{27}C_3.$

63. (a) : For $n > 1$, we have

$(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \ldots {}^nC_nx^n$

$\Rightarrow\ (1 + x)^n = 1 + nx + ({}^nC_2x^2 + {}^nC_3x^3 + \ldots {}^nC_nx^n)$

$\Rightarrow\ (1 + x)^n - 1 - nx = x^2({}^nC_2 + {}^nC_3x + {}^nC_4x^2 + \ldots . {}^nC_nx^{n-2})$

64. (a) : $A = \begin{bmatrix} 0 & 3 \\ -7 & 5 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} -21 & 15 \\ -35 & 4 \end{bmatrix}$

Now, $A^2 - 5A = \begin{bmatrix} -21 & 0 \\ 0 & -21 \end{bmatrix} = -21I$

$\Rightarrow\ 5A - 21I = A^2$

So, $k = 1$.

65. (d) : The foot of perpendicular from $(1, -5, -10)$ to the plane is given by

$\frac{x-1}{1} = \frac{y+5}{-1} = \frac{z+10}{1} = -\frac{(1+5-10-5)}{1+1+1} = 3$

$\therefore\ x = 4, y = -8, z = -7 \Rightarrow (a, b, c) = (4, -8, -7)$

So, $a + b + c = 4 - 8 - 7 = -11$.

66. (c) : $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0} \frac{\sin(a+1)x + \sin x}{x}$ $\quad\left(\frac{0}{0}\text{ form}\right)$

$= \lim\limits_{x\to 0} (a+1)\cos(a+1)x + \cos x$

$= a + 2$ (Using L.H. Rule)

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0} \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}}$

$= \lim\limits_{x\to 0} \frac{\sqrt{1+bx} - 1}{bx} = \lim\limits_{x\to 0} \frac{1}{\sqrt{1+bx}+1} = \frac{1}{2}$

Now, $f(x)$ is continuous at $x = 0$.

$\therefore\ \lim\limits_{x\to 0^-} f(x) = f(0) = \lim\limits_{x\to 0^+} f(x)$

$\Rightarrow\ a + 2 = c = \frac{1}{2} \Rightarrow a = -\frac{3}{2}$ and $a + c = -1$.

67. (b) : $f(2) = f(1 + 1) = f(1) \times f(1) = 2^2$,

$f(3) = f(2 + 1) = f(2) \times f(1) = 2^2 \times 2 = 2^3$

$\therefore\ f(n) = 2^n$

Now, $16(2^n - 1) = \sum\limits_{r=1}^{n} f(a+r) = \sum\limits_{r=1}^{n} 2^{a+r}$

$= 2^a(2 + 2^2 + \ldots . + 2^n) = 2^a \cdot 2\left(\frac{2^n - 1}{2-1}\right)$

$= 2^{a+1}(2^n - 1)$

$\therefore\ 2^{a+1} = 2^4$ or $a = 3$.

68. (b) : Let d be the common difference of the A.P.

Now, $a_6 + a_9 + a_{12} + a_{15} = 20$ (Given)

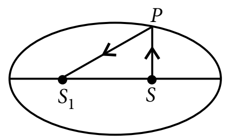
$\Rightarrow\ a_1 + 5d + a_1 + 8d + a_1 + 11d + a_1 + 14d = 20$

$\Rightarrow\ 4a_1 + 38d = 20 \Rightarrow 2a_1 + 19d = 10$

Thus, $S_{20} = \frac{20}{2}[2a_1 + 19d] = 10 \times 10 = 100.$

69. (b) : The contrapositive of "If $x \in A \cap B$, then $x \in A$ and $x \in B$" is "If $x \notin A$ or $x \notin B$, then $x \notin A \cap B$".

70. (b) : The foci are $S_1(-3, 0)$, $S(3, 0)$



The line $x = 3$ intersects the ellipse at

$\frac{y^2}{16} = 1 - \frac{9}{25} = \frac{16}{25},\ y = \frac{16}{5}$

The ray from $S(3, 0)$ to $P\left(3, \frac{16}{5}\right)$ is reflected at P and passes through $S_1(-3, 0)$.

$\therefore$ The slope of $PS_1 = \frac{16}{5} \times \frac{1}{(3+3)} = \frac{8}{15}.$

71. (1) : $\lim\limits_{x\to 0}\left[\frac{x}{\sqrt{1+x} - \sqrt{1-x}}\right]$

$= \lim\limits_{x\to 0}\left[\frac{x(\sqrt{1+x} + \sqrt{1-x})}{1+x-1+x}\right]$

$= \lim\limits_{x\to 0}\left[\frac{x(\sqrt{1+x} + \sqrt{1-x})}{2x}\right] = \frac{1+1}{2} = 1.$

72. (0) : Since, $1 + \sin x \sin^2\frac{x}{2} = 0$

$\therefore\ 1 + \sin x\left(\frac{1-\cos x}{2}\right) = 0$

$\Rightarrow\ 2 + \sin x - \sin x \cos x = 0$

$\Rightarrow\ \sin 2x - 2\sin x = 4$

which is not possible for any x in $[-\pi, \pi]$.

73. (64) : We know that $A(\text{adj } A) = |A|I$

$\Rightarrow\ A \cdot \text{adj } A = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$

$\therefore\ |A| \cdot |\text{adj } A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$

$\Rightarrow\ |A||\text{adj } A| = |A|^3$

$\Rightarrow\ |\text{adj } A| = |A|^2 = 8^2 = 64$ $\quad(\because\ |A| = 8)$

74. (3) : Let $f(x) = x^2 - 2(4k - 1)x + 15k^2 - 2k - 7$, then $f(x) > 0 \Rightarrow D < 0$ $(\because$ coeff. of $x^2 > 0)$

$\Rightarrow 4(4k - 1)^2 - 4(15k^2 - 2k - 7) < 0$

$\Rightarrow k^2 - 6k + 8 < 0 \Rightarrow 2 < k < 4 \Rightarrow k = 3$

75. (5) : No. of items = 100, Incorrect mean $(\bar{x})$ = 40, Incorrect S.D. = 5.1, Incorrect item = 50, Correct item = 40

Now, $\bar{x} = \dfrac{\Sigma x}{n} \Rightarrow 40 = \dfrac{\text{Incorrect } \Sigma x}{100}$

$\therefore$ Incorrect $\Sigma x = 4000$

$\therefore$ Correct $\Sigma x = 4000 - 50 + 40 = 3990$

$\therefore$ Correct mean $= \dfrac{3990}{100} = 39.9$

Now, S.D. $= \sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - (\bar{x})^2}$

$\therefore$ Incorrect $\Sigma x^2 = 162601$

$\therefore$ Correct $\Sigma x^2 = 162601 - (50)^2 + (40)^2 = 161701$

$\therefore$ Correct S.D. $= \sqrt{\dfrac{161701}{100} - (39.9)^2}$

$= \sqrt{1617.01 - 1592.01} = 5$