Solutions

PHYSICS

1. (a) : From the given graphs, as tan37º = 3/4,

$$\vec{F}=\left(\frac{3}{4}x+10\right)\hat{i}+\left(20-\frac{4}{3}y\right)\hat{j}+\left(\frac{4}{3}z-16\right)\hat{k}$$

$$W=\int\vec{F}\cdot d\vec{s}$$

$$=\int_{(0,5,12)}^{(4,20,0)}\left[\left(\frac{3}{4}x+10\right)\hat{i}+\left(20-\frac{4}{3}y\right)\hat{j}+\left(\frac{4}{3}z-16\right)\hat{k}\right]\cdot[dx\hat{i}+dy\hat{j}+dz\hat{k}]$$

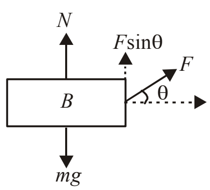
= 192 J.

2. (a) : $-\dfrac{GMm}{R+h}+\dfrac{1}{2}m\left(\dfrac{v_e}{2}\right)^2=0$

$\Rightarrow -\dfrac{GMm}{R+h}+\dfrac{2GMm}{8R}=0 \Rightarrow \dfrac{1}{R+h}=\dfrac{1}{4R}\Rightarrow h=3R$.

3. (c) : Let the body is acted upon by a force at an angle θ with horizontal.

Free body diagram :



$F\cos\theta=\mu(mg-F\sin\theta) \Rightarrow F=\dfrac{\mu mg}{\cos\theta+\mu\sin\theta}$

For minimum force, (cosθ + μsinθ) should be max.

$\Rightarrow -\sin\theta+\mu\cos\theta=0 \Rightarrow \tan\theta=\mu$

or, $\theta=\tan^{-1}(1/\sqrt{3})=30°$.

Substituting, F_{min} = 12.5 kg f.

4. (a) : $f=Rc(Z-1)^2\left(\dfrac{1}{1^2}-\dfrac{1}{2^2}\right)=1.43\times10^{18}$ Hz. .

5. (d) : Given that $m=\dfrac{2.50}{20.0}\text{kg m}^{-1}$, $T=200$ N

The speed of the transverse jerk is given by

$$v=\sqrt{\frac{T}{m}}=\sqrt{\frac{200\times20.0}{2.50}}=\sqrt{1600}=40\text{ ms}^{-1}$$

∴ Time taken by the jerk to reach the other end

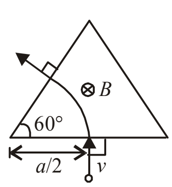
$$=\frac{\text{Distance}}{\text{Speed}}=\frac{20}{40}=0.5\ s.$$

6. (b) : Net charge on system = 0

∴ Net force on system = 0.

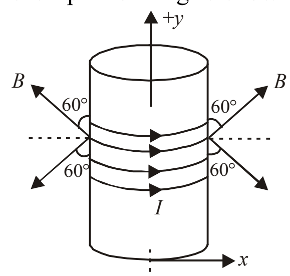
Now consider one charge : $F=qvB$.

7. (b) : The charged particle moves in a circle of radius $a/2$.



$\therefore\quad qvB=\dfrac{mv^2}{a/2}$ or, $B=\dfrac{2mv}{qa}$.

8. (d) : Force on each part of ring is shown in figure.



$F_{resultant}=N\times I\times2\pi rB\sin60°$

$=40\times1\times2\pi(0.5\times10^{-2})\times0.2\times\dfrac{\sqrt{3}}{2}$

$=4\sqrt{3}\pi\times10^{-2}$ N.

Clearly, the force is along $-y$ direction.

9. (d) : The principle can be easily understood from the working of loudspeaker.

10. (b) : $dT=dm(l-x)\omega^2$

$$dT=\frac{m}{l}dx(l-x)\omega^2$$

$$\Rightarrow \int_0^T dT=\int_0^{l/2}\frac{m\omega^2}{l}(l-x)\,dx$$

$$=\frac{m\omega^2}{l}\left[lx-\frac{x^2}{2}\right]_0^{l/2}=\frac{m\omega^2}{l}\left[\frac{l^2}{2}-\frac{l^2}{8}\right]$$

∴ Tension at mid point is : $T=\dfrac{3}{8}ml\omega^2$

$\Rightarrow$ stress $=\dfrac{3ml\omega^2}{8A} \Rightarrow$ strain $=\dfrac{3ml\omega^2}{8AY}$

11. (a) : For a black body, wavelength for maximum intensity:

$\lambda\propto\dfrac{1}{T}$ (Wien's law)

and $P\propto T^4$ (Stefan's law)

$\Rightarrow P\propto\dfrac{1}{\lambda^4} \Rightarrow P'=16P$.

∴ $P'T'=32PT$.

12. (c) : $F=\dfrac{kq^2}{r^2} \Rightarrow k=\dfrac{1}{4\pi\varepsilon_0}$

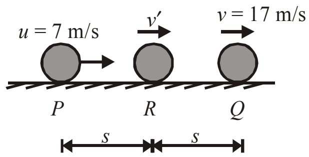
$\Rightarrow \dfrac{kq^2}{r^2}=\dfrac{mv^2}{R_C} \Rightarrow R_C=\dfrac{mv^2r^2}{kq^2}$

$\therefore\quad R_C=\dfrac{4\pi\varepsilon_0v^2r^2m}{q^2}$.

13. (b, d) : Light is propagated in the form of corpuscles according to Newton. However, he was able to get interference (Newton's rings) though he could not explain it.

14. (b, c, d)

15. (a,b,d) :



Let the distance between P and Q be $2s$.

Then $v^2 = u^2 + 2a(2s)$

or, $(17)^2 = (7)^2 + 4as$

$\therefore$ $4as = 240\ \text{m}^2/\text{s}^2$

or, $2as = 120\ \text{m}^2/\text{s}^2$

$\therefore$ velocity of particle at R is

$v'^2 = u^2 + 2as = (7)^2 + (120) = 169$.

$\therefore$ $v' = 13$ m/s.

Now let $t_{PR} = t_1$ and $t_{RQ} = t_2$

Then, $v' = u + at_1$ or, $13 = 7 + at_1$

or, $at_1 = 6$ m/s ...(1)

Similarly, $v = v' + at_2$ or, $17 = 13 + at_2$

or, $at_2 = 4$ m/s ...(2)

From (1) and (2), $\dfrac{t_1}{t_2} = \dfrac{6}{4} = \dfrac{3}{2}$

Average velocity between R and Q is

$$< v_{RQ} > = \frac{s}{t_2} = \frac{v't_2 + \frac{1}{2}at_2^2}{t_2} = v' + \frac{1}{2}at_2$$

$$= 13 + \frac{1}{2}(4) = 15 \text{ m/s}$$

Similarly average velocity between P and R is

$$< v_{PR} > = \frac{s}{t_1} = \frac{ut_1 + \frac{1}{2}at_1^2}{t_1} = u + \frac{1}{2}at_1$$

$$= (7) + \frac{1}{2}(6) = 10 \text{ m/s}$$

16. (a, d) : The acceleration of the bead down the wire is $g\cos\theta$ and the length of wire is $2R\cos\theta$, where R is radius of circle

$\therefore$ $v = \sqrt{2as} = \sqrt{2(g\cos\theta)(2R\cos\theta)}$

$= 2\sqrt{gR}\cos\theta$

i.e., $v \propto \cos\theta$

Further $t = \dfrac{v}{a} = \dfrac{2\sqrt{gR}\cos\theta}{g\cos\theta} = 2\sqrt{\dfrac{R}{g}}$

i.e. t is independent of θ.

17. (a, d) : In uniformly accelerated motion, $v = u + at$

and $v^2 = u^2 + 2as$ or, $v = \sqrt{u^2 + 2as}$

$\therefore$ Power $P = F \cdot v = F(u + at)$

also $P = F\sqrt{u^2 + 2as}$

i.e., power varies linearly with time and parabolically with displacement.

18. (c, d) : Let m be the mass of the block

Initial elongation of the spring will be $x_i = \dfrac{mg}{k}$ ... (1)

When the force F is applied, work done by F and gravity is used to increase the elastic potential energy of spring. Hence,

$(F + mg)x_0 = \dfrac{1}{2}k(x_i + x_0)^2 - \dfrac{1}{2}kx_i^2$ ... (2)

From equations (1) and (2) we get $x_0 = \dfrac{2F}{k}$

Hence, (c) is correct, work done by applied force $F = Fx_0$. Hence (d) is also correct.

19. (a, b, d)

20. (b, c) : $g = \dfrac{GM}{R^2} = \dfrac{G\left(\frac{4}{3}\pi R^3\rho\right)}{R^2} \Rightarrow g \propto R$.

$\dfrac{g_1}{g_2} = \dfrac{R_1}{R_2} = 2$ [$\therefore$ (a) is not true]

$\dfrac{g_1}{g_3} = \dfrac{R_1}{R_3} = 3$ [$\therefore$ (b) is true]

$v = \sqrt{2gR}$

$\therefore$ $\dfrac{V_1}{V_2} = \dfrac{R_1}{R_2} = 2$ [$\therefore$ (c) is true]

$\dfrac{V_1}{V_3} = \dfrac{R_1}{R_3} = 3$ [$\therefore$ (d) is not true].

21. (a) : Angular momentum
= moment of inertia × angular velocity.

i.e., $L = I\omega$.

Differentiating both side w.r.t. time we get, $\dfrac{dL}{dt} = I\dfrac{d\omega}{dt} = I\alpha$.

But, torque $\tau = I\alpha$.

$\therefore \tau = \dfrac{dL}{dt}$ *i.e.*, torque is the rate of change of angular momentum.

22. (c) : For diffraction of a wave, size of an obstacle or aperture should be comparable to the size of wavelength of the wave. As wavelength of light is of the order of 10^{-6} m and obstacle / aperture of this size are rare, therefore, diffraction is not common in light waves. On the contrary, wavelength of sound is of the order of 1 m and obstacle / aperture of this size are readily available, therefore, diffraction is common in sound.

23. (d) : For given V, I is greater for T_1. Therefore $R = \dfrac{V}{I}$ is smaller for T_1 *i.e.* $R_1 < R_2$.

Further in case of conductor, the resistance increase with rise of temperature, $T_2 > T_1$.

24. (b) : In a voltmeter, a high resistance is connected in series with a galvanometer. That is why resistance of voltmeter is highest. In an ammeter, a low resistance is connected in parallel with a galvanometer. That is why resistance of ammeter is lowest.

25. (c) : Heat given : $\Delta Q = n_1C_{V_1}\Delta T \rightarrow$ for gas A

and for gas B, $\Delta Q = n_2C_{V_2}\Delta T$

($\because$ For same heat given, temperature rises by same value for both the gases)

$\Rightarrow$ $n_1C_{V_1} = n_2C_{V_2}$ ... (i)

Also, $(\Delta P_B)V = n_2R\Delta T$ and $(\Delta P_A)V = n_1R\Delta T$

$$\Rightarrow \frac{n_1}{n_2}=\frac{\Delta P_A}{\Delta P_B}=\frac{2.5}{1.5}=\frac{5}{3} \Rightarrow n_1=\frac{5}{3}n_2$$

Substituting in (i),

$$\frac{5}{3}n_2 C_{V_1}=n_2 C_{V_2} \Rightarrow \frac{C_{V_2}}{C_{V_1}}=\frac{5}{3}=\frac{\left(\frac{5}{2}R\right)}{\left(\frac{3}{2}R\right)}$$

Hence, gas B is diatomic and gas A is monoatomic.

26. (d) : Since $n_1=\frac{5}{3}n_2$, therefore $\frac{125}{M_A}=\frac{5}{3}\left(\frac{60}{M_B}\right)$

(From experiment 1 : $W_A = 225$ g and $W_B = 160$ g)

$\Rightarrow 5M_B = 4M_A$

The above relation holds for the pair -

Gas A : Ar and gas B : O_2.

27. (d) : Number of molecules in $A = nN_A$

$$=\frac{125}{40}N_A=3.125\,N_A$$

(Since $n = 125/40$ for A)

28. (c) : Internal energy at any temperature, T

$$U_i = nC_V T = \left(\frac{125}{40}\right)\left(\frac{3R}{2}\right)(300)$$

[$\because C_V$ for monoatomic gas = $3R/2$]

$\Rightarrow U_i = 2782.3$ cal.

29. (b) : $n_A C_{V_A} \times 300 + n_B C_{V_B} \times 300 = n_A C_{V_A} T + n_B C_{V_B} T$

$\Rightarrow T = 300$ K.

(It could also be seen directly that temperature finally will be 300 K, since no heat exchange take place between those gases as their initial temperatures are same).

Since volume remains same but number of moles increases, therefore pressure increases.

30. (c) : Time constant, $\tau=\frac{L}{R}=\frac{50\text{ mH}}{10}=5\text{ ms}$

Growth equation in L-R circuit is $i=i_0\left(1-e^{-\frac{R}{L}t}\right)$

or, $\frac{i_0}{2}=i_0\left(1-e^{-\frac{R}{L}t}\right) \Rightarrow \frac{1}{2}=1-e^{-\frac{R}{L}t}$

or, $e^{-\frac{R}{L}t}=\frac{1}{2}$ or, $e^{\frac{R}{L}t}=2 \Rightarrow \frac{R}{L}t=\ln 2$

$\therefore t=\frac{L}{R}\ln 2=5\times10^{-3}\times0.693 \Rightarrow t=3.5$ ms.

31. (a) : Current in L-R circuit is given as

$i = i_0(1 - e^{-t/\tau})$

where $i_0 = E/R$ and $\tau = L/R$

$$Q=\int_0^\tau i\,dt=i_0\int_0^\tau(1-e^{-t/\tau})\,dt=i_0\left[t-\frac{e^{-t/\tau}}{-1/\tau}\right]_0^t=\frac{i_0\tau}{e}.$$

32. (b) : $i = i_0 e^{-t/\tau}$

Here, $i = i_0/\eta$ or, $\frac{i_0}{\eta}=i_0 e^{-t_0/\tau} \Rightarrow \frac{1}{\eta}=e^{-t_0/\tau}$

or, $\eta=e^{t_0/\tau} \Rightarrow \frac{t_0}{\tau}=\ln\eta$ or, $\tau=\frac{t_0}{\ln\eta}$.

33. (a) : Induced emf across inductor,

$e = E - IR$

Hence graph of e *versus* i is a straight line with positive intercept and negative slope.

34. (b) : The inductors are in parallel. Therefore potential difference across them is same. Hence $V_1 = V_2$.

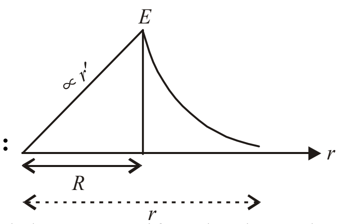
or, $L_1\left(\frac{di_1}{dt}\right)=L_2\left(\frac{di_2}{dt}\right)$ or, $L_1 di_1 = L_2 di_2$

After integrating, $L_1 i_1 = L_2 i_2$

or, $\frac{i_1}{i_2}=\frac{L_2}{L_1}$.

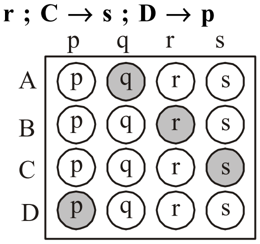
35. (d) : The electric field for a current carrying conductor is along the direction of the current in the conductor. The magnetic induction is along the plane perpendicular to r and perpendicular to the conductor. Hence the cross product of i and $\vec{r}$ in the passage. $\vec{B}$ is along a circular path with the conductor as the axis,

36. (b) :

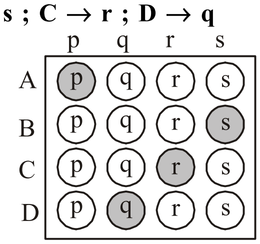


Field due to a uniformly charged solid sphere.

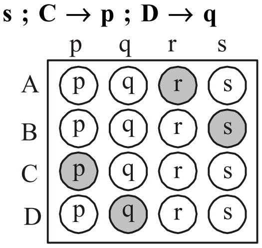
37. A → q ; B → r ; C → s ; D → p



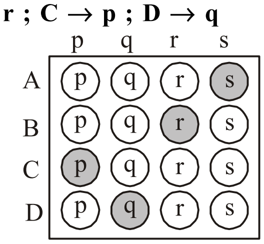
38. A → p ; B → s ; C → r ; D → q



39. A → r ; B → s ; C → p ; D → q



40. A → s ; B → r ; C → p ; D → q



41. **(2) :** $I_1\omega_{01} + I_2\omega_{02} = (I_1 + I_2)\omega$

$(5\ \text{kgm}^2)\cdot(2\pi\cdot 10\ \text{rps}) + 20\ \text{kgm}^2(0)$

$= (5 + 20)\ \text{kgm}^2 .\ (2\pi\upsilon)$

$$\Rightarrow \quad \upsilon = \frac{5\times 10}{25} = 2\ \text{rps}.$$

42. **(5) :** This is an equation of a stationary wave. The coefficient of x represents k.

$$\Rightarrow k = \frac{2\pi}{\lambda} = 1.57 \text{ or } \lambda = \frac{2(3.14)}{1.57} = 4\ \text{cm}$$

Since each loop in a stationary wave exists between consecutive nodes, the length of the loop is $\frac{\lambda}{2}$ or 2 cm.

$$l = n\left(\frac{\lambda}{2}\right) \text{ or } n = \frac{2l}{\lambda} = \frac{2(10\ \text{cm})}{4\ \text{cm}} = 5.$$

43. **(3) :** Consider dN number of turns of radius r and thickness dr. Let dE be the corresponding induced *emf*, then

$$dE = (dN)\cdot\left(\frac{d\phi}{dt}\right)$$

$$dE = dN\cdot\frac{d}{dt}(\pi r^2\cdot B_0 \sin \omega t)$$

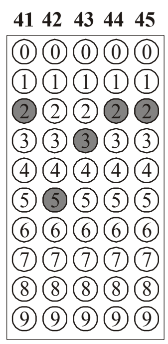
$$dE = \left(\frac{N}{a}\right)dr\ (\pi r^2\omega\cdot B_0 \cos \omega t)$$

$$E = \int dE = \left(\frac{N\pi\cdot\omega B_0 \cos\omega t}{a}\right)\int_0^a r^2 dr$$

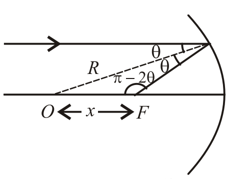
$$= \frac{N\pi\omega(B_0\cos\omega t)a^3}{3a}$$

$$E_{\max} = \frac{\pi N a^2 B_0\ \omega}{3}.$$

$\therefore \quad n = 3.$



44. **(2) :**



Using sin θ law, $\frac{x}{\sin\theta} = \frac{R}{\sin(\pi - 2\theta)}$

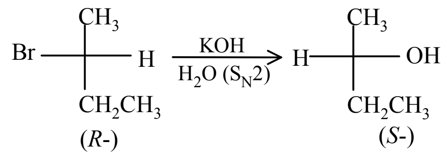
$$\Rightarrow x = \frac{R\sin\theta}{\sin 2\theta} = \frac{R\sin\theta}{2\sin\theta\cos\theta} \Rightarrow x = \frac{R}{2\cos\theta}.$$

Here $m = 2$.

45. **(2) :** For positronium atom we can use reduced mass concept, $\mu = \frac{m\times m}{2m} = \frac{m}{2}$ where m is mass of electron. Thus, the energy levels of a positronium atoms are given by, $E'_n = \frac{\mu}{m}\times E_n,\ E'_n = \frac{E_n}{2}$ where E_n is the energy of n^{th} energy state of hydrogen atom. Thus, wavelength of positronium spectral lines is double (two times) of that hydrogen spectra.

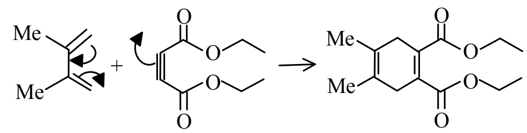
CHEMISTRY

1. **(b) :**



S is formed due to S_N2 inversion.

2. **(b) :** Given reaction is an example of Diels-Alder reaction which is the conjugate addition of a diene and dienophile.



3. **(a) :** $2HSO_4^- \rightarrow S_2O_8^{--} + 2H^+ + 2e^-$

So required rate = 1 mol/hr = 2 mols of e^-/hr

$$= \frac{2\times 96500\ \text{C}}{3600\ \text{sec}} = \frac{2\times 965}{36}\text{A} \approx 53.6\ \text{A}.$$

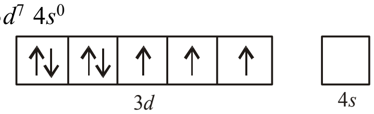
So required current = $\frac{4}{3}\times 53.6\ \text{A} = 71.47\ \text{A}.$

4. **(a) :** $Ca_3P_2 + 6H_2O \rightarrow 3Ca(OH)_2 + 2PH_3$

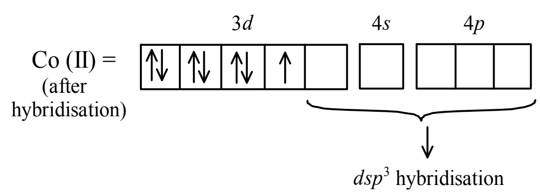
5. **(d) :** $\alpha = \frac{i-1}{(n-1)} \Rightarrow \frac{4-1}{(n-1)} = \frac{3}{4}$

So, $n = 5$, hence given complex must dissociate into five ions $\equiv Ba_3[Co(CN)_5]_2$.

Paramagnetic moment indicates one unpaired electron Co(II) $= 3d^7\ 4s^0$

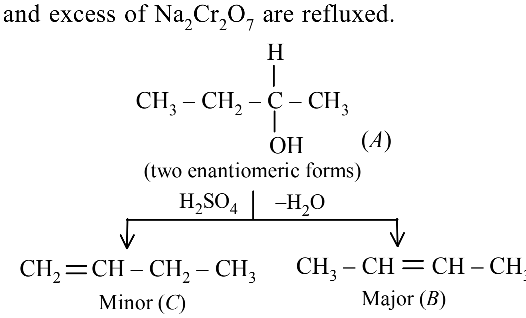


Before hybridisation



6. **(c) :** The maximum percentage yield of butanol is obtained from 3 as the butanol formed is distilled out at a lower temperature. The maximum % yield of butanoic acid is obtained in 2 where 1-butanol and excess of $Na_2Cr_2O_7$ are refluxed.

7. **(b) :**



(B) will give (A) again. Addition in (C) will occur against Markownikoff's rule. Hence (C) will give isomer of A *i.e.* it will form butan-1-ol.

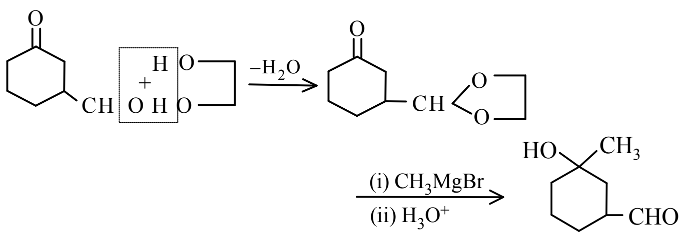
8. **(c)** : In presence of H_2O_2, $FeCl_3$ and $K_3Fe(CN)_6$ gives blue colloidal solution.

9. **(a) :** Carbon atoms are at corners and are at alternate corners. So from geometry,

$$\sqrt{3}\left(\frac{a}{2}\right)\frac{1}{2} = 2r$$

So required ratio $= \dfrac{2r}{a} = \dfrac{\sqrt{3}}{4} = \sqrt{\dfrac{3}{16}}.$

10. **(b) :** Aldehyde is more reactive than ketone in dioxalin formation, so aldehyde group is protected.



11. **(c) :** $C_{rms} = \sqrt{\dfrac{3RT}{M}}$

$$\frac{C_{rms}(H_2)}{C_{rms}(N_2)} = \sqrt{\frac{T(H_2)}{M(H_2)} \times \frac{M(N_2)}{T(N_2)}}$$

$$\sqrt{7} = \sqrt{\frac{T(H_2)}{T(N_2)} \times \frac{28}{2}} \text{ or, } \frac{T(H_2)}{T(N_2)} = \frac{1}{2}$$

12. **(d) :** ClO_3^- (pyramidal) | NO_3^- (planar)

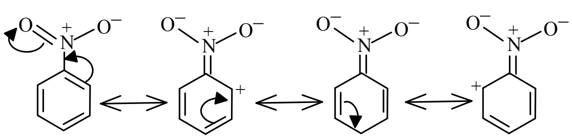
$\therefore$ cannot be isomorphous.

13. **(b, d) :** If two groups or atoms present on one of the two doubly bonded C-atoms are similar then that type of alkene does not show *cis-trans* isomerism.

14. **(a, b, c) :** These are the properties of graphite.

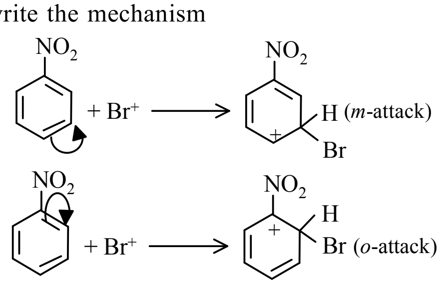
15. **(b, c) :** $Al_2(CH_3)_6$ and B_2H_6 both have three centre two electron bonds.

16. **(a, b) :** Resonance in nitrobenzene

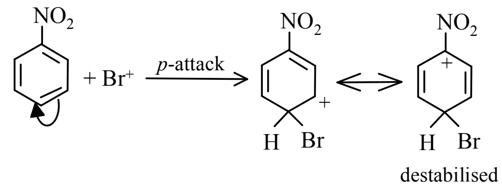


I II III IV

Nitro-group because of electron withdrawing nature reduces electron density more at *o*- and *p*-positions than at *m*-position. If we write the mechanism



(destabilised because +ve charge is on the carbon attached to electron withdrawing group).



Thus, the intermediate carbocation formed after the initial attack of Br^+ at the *meta*-positions is least destabilised.

17. **(a, b, c, d) :**

(a) Pyrolysis of esters, *syn*-1,2-elimination.
(b) Cope's elimination of trialkyl amine oxides.
(c) Hofmann elimination of quarternary ammonium hydroxide.
(d) Hofmann elimination of sulphonium hydroxide.

18. **(a, b, c, d) :** $H - C \equiv N$; $N \equiv C - C \equiv N$; $O = C = C = C = O$; $O = C = O$

19. **(a, b) :** (a) $CF_3I + OH^- \longrightarrow CF_3^- + IOH$

$$CF_3^- \longrightarrow CF_3H + IO^-$$

(b) $Rb[ICl_2] \xrightarrow{\Delta} RbCl + ICl$, RbCl is more stable due to high lattice energy so will be the product.

(c) polymeric silicones are formed.

20. **(a, b) :** For $AlCl_3 \rightleftharpoons Al^{3+} + 3Cl^-$

$\therefore$ $K_{sp} = S \times (3S)^3 = 27S^4$

21. **(c) :** Reactions of higher order are rare because chances for larger number of molecules to come simultaneously for collision are less.

22. **(d) :** In actual practice transition metals react with acid very slowly and act as poor reducing agents. This is due to the protection of metal as a result of formation of thin oxide protective film. Further, their poor tendency as reducing agent is due to high ionisation energy, high heat of vapourization and low heat of hydration.

23. **(b) :** Both carbanions (formed in presence of base) and enol form (formed in presence of an acid) act as nucleophiles and hence add on the carbonyl group of aldehydes and ketones to give aldols.

24. **(a) :** The loss of one α-particle will reduce the mass number by four and atomic number by two. Subsequent two β-emissions will increase the atomic number by two without affecting the mass number. Hence, the new element will be only an isotope of the parent nucleide and hence its position in the periodic table remains unchanged.

25. **(b) :** $u_{r.m.s} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3P}{d}}$

Number of molecules $= 2 \times 10^{21}$

$\because$ Mass of 6.023×10^{23} molecules $= 14 \times 2 = 28$ g

$\therefore$ Mass of 2×10^{21} molecules $= \dfrac{28 \times 2 \times 10^{21}}{6.023 \times 10^{23}} = 0.093$ g

Density, $d = \dfrac{0.093}{1} = 0.093\ \text{gL}^{-1}$

$$= \frac{0.093\times10^{-3}\ \text{kg}}{10^{-3}\ \text{m}^3} = 0.093\ \text{kg m}^{-3}.$$

$$u_{\text{rms}} = \sqrt{\frac{3\times7.57\times10^3}{0.093}} = 494.16\ \text{ms}^{-1}.$$

26. (c) : $u_{\text{rms}}^2 = \dfrac{3RT}{M} = \dfrac{3P}{d}$

$\therefore\ \dfrac{RT}{M} = \dfrac{P}{d} = \dfrac{7.57\times10^3}{0.093}$

or, $T = \dfrac{7.57\times10^3}{0.093}\times\dfrac{28\times10^{-3}}{8.314} = 274.13\ \text{K}.$

27. (b) : $\dfrac{\text{most probable speed}}{u_{\text{rms}}} = 0.82$

$\therefore$ Most probable speed = 0.82 × 494.16 = 405.2 ms^{-1}.

28. (c) : BF_3 is the weakest Lewis acid because it is less electron deficient due to back donation or back bonding of electron from F atom.

As a result of back donation of electron from fluorine to boron, the electron deficiency of boron is reduced and Lewis acid character is decreased. The tendency for the back bonding ($p\pi$-$p\pi$ bonds) is maximum in BF_3 and decreases rapidly from BF_3 to BI_3.

29. (b) : Some of the total BF_3 can combine with HF formed during hydrolysis to form HBF_4.

30. (d) : Since BCl_3 can accept only one lone pair hence $Cl_3B(C_5H_5N)_2$ is not possible.

31. (d) : Reaction of BCl_3 with $LiAlH_4$ produces B_2H_6 which contains two 3-centred-2-electron bonds and four 2-centred-2-electron bonds.

32. (a) : Given, mixture of (A) and (B) $\xrightarrow[\text{+ KOH (aq.)}]{CHCl_3}$

organic layer (A) + alkaline aqueous layer (B)

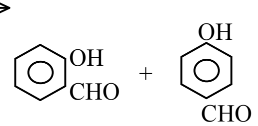
Organic layer on treating with KOH (alc.) produces (C_7H_5N) (C) of unpleasant odour and thus (C) is C_6H_5NC. Therefore, (A) is $C_6H_5NH_2$.

33. (b) : Carbylamine reaction

$$\underset{\text{(aniline) }(A)}{C_6H_5NH_2} + CHCl_3 + 3KOH\ (\text{alc.}) \rightarrow \underset{\text{phenyl isocyanide }(C)}{C_6H_5NC} + 3KCl + 3H_2O$$

34. (d) : Alkaline layer on treating with $CHCl_3$ followed by acidification gives two isomers having formula ($C_7H_6O_2$). This is Reimer-Tiemann reaction and thus (B) is C_6H_5OH.

$$\underset{\text{phenol }(B)}{C_6H_5OH} + CHCl_3 + KOH \xrightarrow{H^+}$$



o-hydroxy benzaldehyde + p-hydroxy benzaldehyde

35. (d) : The increase in mass at the cathode is due to deposition of Cu($Cu^{2+} + 2e^- \rightarrow Cu$). The loss in mass of anode is due to loss of Cu and Fe because of their oxidation. These two are active metals and will oxidise as

$Cu \rightarrow Cu^{2+} + 2e^-$

$Fe \rightarrow Fe^{2+} + 2e^-$

and loss of Ag and Au to fall in anode mud.

Thus, gain in weight at cathode is due to deposition of Cu = 22.011 g

$\therefore$ Moles of Cu deposited at cathode = $\dfrac{22.011}{63.5}$ = 0.3466 g

Equivalent of Cu and Fe dissolved at anode

$$= \frac{I\cdot t}{96500} = \frac{140\times482.5}{96500} = 0.70$$

$\therefore$ Moles of Cu and Fe dissolved at anode = $\dfrac{0.70}{2} = 0.35$

(both Cu and Fe are bivalent losing two electrons).

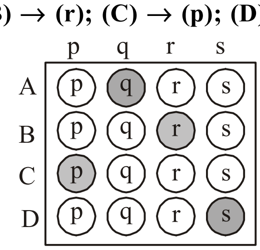
Moles of Fe dissolved at anode = 0.3500 – 0.3466 = 0.0034

$\therefore$ Wt. of Fe dissolved at anode = 0.0034 × 56 = 0.190 g.

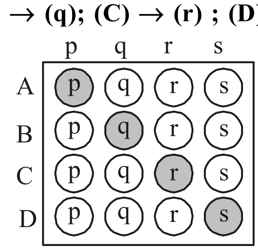
36. (c) : Anode weight loss of 22.260 g contains 22.011 g Cu

$\therefore$ % Cu originally present = $\dfrac{22.011}{22.26}\times100 = 98.88\%.$

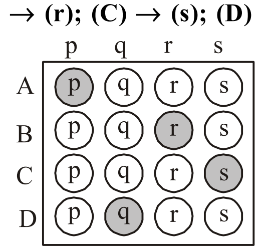
37. (A) → (q) ; (B) → (r); (C) → (p); (D) → (s)



38. (A) → (p); (B) → (q); (C) → (r) ; (D) → (s)



39. (A) → (p); (B) → (r); (C) → (s); (D) → (q)



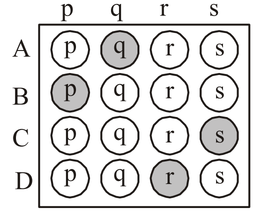
(A) $ArOCH_3 + HI \xrightarrow{100°C} ArOH + CH_3I$

(B) $R_2O + BF_3 \longrightarrow [R_2O]^+[BF_3]^-$

(C) $R_2O + H_2SO_4 \longrightarrow [R_2OH]^+HSO_4^-$

(D) $PhOH + (C_2H_5)_2SO_4 \xrightarrow{NaOH} PhOC_2H_5 + C_2H_5NaSO_4$

40. (A) → (q); (B) → (p); (C) → (s); (D) → (r)



41. (4) : Kinetic energy = (1/2) mv^2

$0.0327 \times 1.602 \times 10^{-19} = (1/2) \times 1.675 \times 10^{-27} \times v^2$

(1 eV = 1.602×10^{-19} J)

$\therefore\ v = 2500.0$ m/sec =2.50 km/sec

Thus time taken to move 10 km = $\dfrac{10}{2.5} = 4.0$ sec

Now, neutrons left (N) after 4.0 sec can be obtained by

$$\lambda = \frac{2.303}{t}\log\frac{N_0}{N}\ ;\ \frac{0.693}{700} = \frac{2.303}{4}\log\frac{N_0}{N}$$

$$\frac{N_0}{N} = 1.004$$

$\therefore\ N = 99.60$ % $\quad (N_0 = 100)$

$\therefore$ Number of neutrons decayed = 0.4 %

or $\quad \dfrac{4}{10} = 0.4$, so value of x is 4.

42. (5) : $\lambda_2 = 30.4$ nm $= 30.4 \times 10^{-7}$ cm

$\lambda_1 = 108.5$ nm $= 108.5 \times 10^{-7}$ cm

Suppose the excited state be n_2. The electron falls first from n_2 to n_1 and then n_1 to ground state.

$$\frac{1}{\lambda} = Z^2R_H\left[\frac{1}{1^2} - \frac{1}{n_1^2}\right]$$

$$\frac{1}{30.4\times10^{-7}} = 2^2 \times 109678\left[\frac{1}{1^2} - \frac{1}{n_1^2}\right]$$

$$\frac{1}{n_1^2} = 1 - \frac{1}{30.4\times10^{-7}\times4\times109678} = 1 - 0.75 = 0.25$$

$$n_1^2 = \frac{1}{0.25} = 4\ ;\ n_1 = 2$$

$$\frac{1}{\lambda_1} = Z^2R_H\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{108.5\times10^{-7}} = 4\times109678\left[\frac{1}{4} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{n_2^2} = \frac{1}{4} - \frac{1}{108.5\times10^{-7}\times4\times109678} = 0.25 - 0.21 = 0.04$$

$$n_2^2 = \frac{1}{0.04} = 25\ ;\ \therefore\ n_2 = 5$$

43. (0) : Since the crystalline compound(B) dissolves in hot water and gives a yellow precipitate with NaI, it should be lead chloride, $PbCl_2$ and the solution (A) consists of a lead salt.

$$\underset{(B)}{PbCl_2} + 2NaI \rightarrow \underset{(D)}{PbI_2} + 2NaCl$$

The compound (A) does not give any gas with dilute HCl but liberates a reddish brown gas on heating, it should be lead nitrate, $Pb(NO_3)_2$

$$\underset{(A)}{2Pb(NO_3)_2} \rightarrow 2PbO + \underset{\text{Reddish brown gas}}{4NO_2} + O_2$$

Lead chloride is sparingly soluble in water. When H_2S is passed, it gives a black precipitate of lead sulphide, PbS.

$$PbCl_2 + H_2S \rightarrow \underset{\substack{\text{Black}\\(C)}}{PbS} + 2HCl$$

Thus, (A) is lead nitrate, $Pb(NO_3)_2$,

(B) is lead chloride, $PbCl_2$,

(C) is lead sulphide, PbS,

(D) is lead iodide, PbI_2.

In $Pb(NO_3)_2$ oxidation state of Pb is +2.

In $PbCl_2$ oxidation state of Pb is +2.

So, change in O.S is zero.

44. (3) Burnt plaster :

$$CaSO_4 \xrightarrow{\Delta} O_2 + SO_2 + CaO$$

Number of compounds =3

45. (2) (1)(X), (C_5H_8O) does not react with Lucas reagent appreciably at room temperature but gives precipitate with ammoniacal $AgNO_3$,and thus, (X) has terminal alkyne linkage as well as primary alcoholic group.

(2) (X) on hydrogenation and then reacting with HI gives n-pentane and thus, (X) is a straight chain compound.

(3) Keeping in view of the above facts (X) may be

$$\underset{(X)\quad\text{Pent-4-yn-1-ol}}{HOCH_2-CH_2-CH_2-C\equiv CH}$$

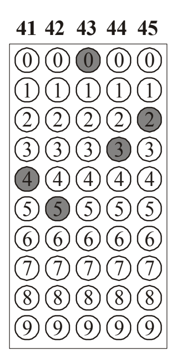
(4) Its reaction with MeMgBr gives CH_4. (It has two acidic or active H atoms) and thus, 1 mole of (X) will give two moles of CH_4.

$$OHCH_2\cdot CH_2CH_2\cdot C\equiv CH + 2CH_3MgBr \rightarrow 2CH_4$$

$\because$ 84 g (X) gives 2×22.4 litre CH_4

$\therefore$ 0.42 g (X) will give $\dfrac{2\times22.4\times0.42}{84} = 224$ ml CH_4

Thus compound (X) has 2 acidic H-atoms.



MATHEMATICS

1. **(b) :** Let two numbers be a and b

$$x\cdot\frac{a+b}{2}=y\cdot\sqrt{ab}=z\cdot\frac{2ab}{a+b}\quad\Rightarrow\quad\frac{a+b}{2\sqrt{ab}}=\frac{y}{x}=\frac{z}{y}$$

$\Rightarrow$ $y^2 = xz$ $\Rightarrow$ x, y, z are in G.P.

2. **(a) :** $\int_0^3 (x-[x])^{[x]}dx=\int_0^1 dx+\int_1^2 (x-1)dx+\int_2^3 (x-2)^2 dx$

Put $y = x - 1$, $z = x - 2$

$$=1+\int_0^1 y\,dy+\int_0^1 z^2 dz=1+\left[\frac{y^2}{2}\right]_0^1+\left[\frac{z^3}{3}\right]_0^1=1+\frac{1}{2}+\frac{1}{3}=\frac{11}{6}.$$

3. **(b) :** Clearly $x^2 + 4x + \alpha^2 - \alpha \ge 0\ \forall\ x \in R$ and must take all values of the interval $[0, \infty)$

$\Rightarrow$ $D = 0$

i.e. $16 - 4(\alpha^2 - \alpha) = 0$ $\Rightarrow$ $\alpha^2 - \alpha = 4$

$\Rightarrow$ $\alpha=\frac{1\pm\sqrt{17}}{2}.$

4. **(b) :** $2^p + 3^q + 5^r = 2^p + (4 - 1)^q + (4 + 1)^r$

$= 2^p + 4\lambda_1 + (-1)^q + 4\lambda_2 + 1^r$ (λ_1, λ_2 are integers)

If $p = 1$, q should be even and r can be any number. On the other hand if $p \ne 1$, q should be odd and r can be any number.

Total number of ordered triplets

$= 5 \times 10 + 9 \times 5 \times 10 = 500.$

5. **(d) :** $A(\text{Adj } A) = |A|\, I_n$

Clearly, $|A| = 4$. $n = 3$

$|\text{Adj }(\text{Adj } A)| = |A|^{(n-1)^2} = 4^4 = 256$

$|\text{Adj } A| = |A|^{n-1} = 4^2 = 16$

$\therefore$ $\frac{|\text{Adj}(\text{Adj } A)|}{|\text{Adj } A|}=\frac{256}{16}=16.$

6. **(b) :** $e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3}$ lie on a unit circle having centre at origin.

$\therefore$ Circumcentre of ΔPQR is origin, centroid is

$\frac{e^{i\theta_1}+e^{i\theta_2}+e^{i\theta_3}}{3}$

Now, centroid divides orthocentre and circumcentre in the ratio 2 : 1

$\therefore$ Orthocentre is $e^{i\theta_1}+e^{i\theta_2}+e^{i\theta_3}$.

7. **(c) :** ΔABC is an equilateral triangle, centroid of ΔOBC is $-\frac{1}{3}\hat{i}$ and orthocentre is $\hat{i}$ and centroid divides orthocentre and circumcentre in the ratio 2 : 1.

$\therefore$ Circumcentre is $-\hat{i}$.

8. **(a) :**

$$I=\int_0^{\pi/4}[f(x)(\cos x+\sin x)+f'(x)(\sin x-\cos x)]dx$$

$$=\int_0^{\pi/4} f(x)(\sin x+\cos x)dx+\int_0^{\pi/4} f'(x)(\sin x-\cos x)dx$$

$$=f(x)(\sin x-\cos x)\Big|_0^{\pi/4}-\int_0^{\pi/4} f'(x)(\sin x-\cos x)dx+\int_0^{\pi/4} f'(x)(\sin x-\cos x)dx$$

$$=f\left(\frac{\pi}{4}\right)+f(0)=f(0).$$

9. **(d) :** $f(x) = a_1x + a_2x^3 + a_3x^5 + \ldots + a_nx^{2n-1}$

$f'(x) = 3a_2x^2 + 5a_3x^4 + \ldots + (2n - 1)a_nx^{2n-2}$

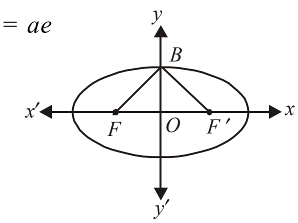
$= x^2[3a_2 + 5a_3x^2 + \ldots + (2n - 1)a_nx^{2n-4}] = 0$ if $x = 0$

$x = 0$ is the only critical point.

$f'(x)$ does not change sign while x crosses 0.

$\therefore$ No extrema.

10. **(b) :** By the given condition, $b = ae$



$\therefore$ $b^2 = a^2(1 - e^2)$

$\Rightarrow$ $a^2e^2 = a^2 - a^2e^2$

$\Rightarrow$ $2e^2=1$ $\Rightarrow$ $e=\frac{1}{2e}.$

11. **(a) :** Consider $f(x) = \sin x - e^{-x}$.

Let $f(x) = 0$ has real roots say α, β.

Rolle's theorem is applicable to $f(x)$ in $[\alpha, \beta]$.

$\therefore$ $f'(x) = 0$ has atleast one root in (α, β).

i.e. $\cos x + e^{-x} = 0$ has at least one root in (α, β)

$\therefore$ $e^x \cos x = -1$ has at least one root in (α, β).

12. **(d) :** Since the normals are perpendicular

$\therefore$ the tangent will also be perpendicular to each other

$\therefore$ they will intersect on the directrix.

Equation of parabola is

$y^2 - 4y + 4 = 2x + 4$ or $(y - 2)^2 = 2(x + 2)$

$\therefore$ Equation of the directrix is

$x + 2 = -1/2$. *i.e.* $2x + 5 = 0$.

13. **(b, d) :** Eccentricity of ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ is

$e=\sqrt{3/4}$

Eccentricity of the ellipse $\frac{x^2}{16}+\frac{y^2}{\lambda^2}=1$ is

$e=\sqrt{\frac{16-\lambda^2}{16}}$

$\therefore$ $\frac{16-\lambda^2}{16}=\frac{3}{4}$ $\Rightarrow$ $\lambda^2=4$ $\Rightarrow$ $\lambda=2$

or, Eccentricity of the ellipse $\frac{x^2}{16}+\frac{y^2}{\lambda^2}=1$ is

$\sqrt{\frac{\lambda^2-16}{\lambda^2}}$ $\therefore$ $\frac{\lambda^2-16}{\lambda^2}=\frac{3}{4}$ *i.e.* $4\lambda^2-64=3\lambda^2$

i.e. $\lambda^2 = 64$ $\Rightarrow$ $\lambda = 8$.

14. **(b, c) :** $AB=\sqrt{(5-1)^2+(6-2)^2}=4\sqrt{2}$

$\therefore$ Ellipse if $\lambda>4\sqrt{2}$

Line segment if $\lambda=4\sqrt{2}$.

15. (a, b, c, d) : $f(x)=\begin{cases}3x^2+12x-1, & -1\le x\le 2\\ 37-x, & 2<x\le 3\end{cases}$

$\lim\limits_{x\to 2^-} f(x)=12+24-1=35 \Rightarrow f(2)=35$

$\lim\limits_{x\to 2^+} f(x)=37-2=35$

$\therefore$ $f(x)$ is continuous at $x = 2$.

$f'(x)=\begin{cases}6x+12 & -1\le x<2\\ -1 & 2<x\le 3\end{cases}$

$\therefore$ $f(x)$ is increasing on $[-1, 2]$

$f(x)$ is continuous on $[-1, 3]$

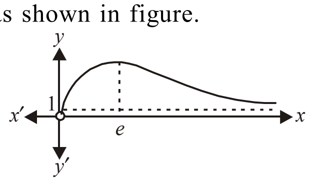
$f'(2^-) = 24, f'(2^+) = -1$

$\therefore$ $f'(2)$ does not exist.

$f(x)$ has the greatest value at $x = 2$.

16. (b, c) : Consider the function $f(x) = x^{1/x}$.

Its graph is as shown in figure.



Clearly, $f(x)$ is increasing for $0 < x < e$ and is decreasing for $x \ge e$.

17. (b, c, d) : For the domain of $(\log(3-x))^{-1}$

$1 \ne 3 - x > 0$

i.e. $x \in (-\infty, 2) \cup (2, 3)$

For the domain of $\cos^{-1}\left(\dfrac{2-|x|}{4}\right)$

$-1\le \dfrac{2-|x|}{4}\le 1$ *i.e.* $-4 \le 2 - |x| \le 4$

i.e. $-6 \le -|x| \le 2$ *i.e.* $-2 \le |x| \le 6$

i.e. $0 \le |x| \le 6 \Rightarrow -6 \le x \le 6$ ($\because |x| \ge 0$)

$\therefore$ The domain is $[-6, 2) \cup (2, 3)$.

18. (b, c) : $g\left(\dfrac{1}{2}\right)=\left[\dfrac{1}{2}[2]\right]=[1]=1;$

$g\left(\dfrac{3}{4}\right)=\left[\dfrac{3}{4}\left[\dfrac{4}{3}\right]\right]=\left[\dfrac{3}{4}\right]=0$

If $1/t = n, n \in N$; $g(t)=\left[t\left(\dfrac{1}{t}\right)\right]=\left[\dfrac{1}{n}[n]\right]=1$

When $1/t = n + h, n \in N$ and $0 < h < 1$

$\therefore$ $g(t)=\left[\dfrac{1}{n+h}[n+h]\right]=\left[\dfrac{n}{n+h}\right]=0$

$\therefore$ $g(t)$ is not continuous at all $t = \dfrac{1}{n}\cdot n\in N$.

19. (a, b, c) : $f(x)=\begin{cases}x^2+2, & x<0\\ 3, & x=0\\ x+2, & x>0\end{cases}$

$f(0)=3,\ \lim\limits_{x\to 0^-} f(x)=2,\ \lim\limits_{x\to 0^+} f(x)=2$

$\therefore$ $f(x)$ has a maximum at $x = 0$

$f'(x) = 2x, x < 0$

$\therefore$ $f'(x) < 0$ for $x < 0$

$\therefore$ $f(x)$ is decreasing on the left of 0

$f''(x) = 2, x < 0$

$\therefore$ $f''(x) > 0, x < 0$

$\therefore$ $f'(x)$ is increasing on the left of 0.

20. (a, c) : $x^2(30 - y)^2 = x^2(x - 30)^2$

Let $f(x) = x^2(x - 30)^2$

$\therefore$ $f'(x) = 2x(x - 30)^2 + 2x^2(x - 30)$

$= 2x(x - 30)(x - 30 + x)$

$= 2x(x - 30)(2x - 30)$

$\therefore$ $x = 15$ and 30 are critical points.

$x = 15$ is a maximum

$x = 30$ is a minimum

$f(15) = 15^4,\ f(30) = 0$

$\lim\limits_{x\to 0} f(x)=0,\quad \lim\limits_{x\to 60} f(x)=60^2\cdot 30^2$

Since $60 \notin$ domain

$\therefore$ There is no greatest value

0 is the least value attained at $x = 30$

21. (a) : $\sin A = \dfrac{3}{\sqrt{13}},\quad \cos B = \dfrac{5}{\sqrt{26}}$

$\cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\dfrac{9}{13}} = \dfrac{-2}{\sqrt{13}}$

($\because$ A is obtuse angle)

$\sin B = \sqrt{1-\dfrac{25}{26}} = \dfrac{1}{\sqrt{26}}$

Since $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$=\dfrac{1}{\sqrt{13}\sqrt{26}}\times(15-2)=\dfrac{\sqrt{13}\,\sqrt{13}}{\sqrt{13}\,\sqrt{13}\,\sqrt{2}}=\dfrac{1}{\sqrt{2}}$

$\therefore$ $A + B = 135°$

$\therefore$ Both statement-1 and statement-2 are true and statement-2 is correct explanation of statement-1.

22. (a) : $\vec{a} = 0,\ \vec{b} = \hat{i}-\hat{j}+\hat{k},\ \vec{c} = 3\hat{j},\ \vec{d} = \hat{i}-\hat{k}$

$\vec{b}\times\vec{d} = \hat{i}+2\hat{j}+\hat{k} \Rightarrow \vec{n}=\dfrac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{6}}$

Shortest distance $= 3\hat{j}\cdot\dfrac{(\hat{i}+2\hat{j}+\hat{k})}{\sqrt{6}} = \sqrt{6}$.

23. (a) : $C_0^2+C_1^2+C_2^2+....+C_n^2 = \dbinom{2n}{n}$

$C_0^2-C_1^2+C_2^2-....+C_n^2 = (-1)^{n/2}\dbinom{n}{n/2}$

$\therefore$ $C_1^2 + C_3^2 + C_5^2 + ... + C_{n-1}^2$

$=\dfrac{1}{2}\left[\dbinom{2n}{n}-(-1)^{n/2}\dbinom{n}{n/2}\right]$

$=\dbinom{2n-1}{n}-(-1)^{n/2}\dbinom{n-1}{n/2}$

24. (d) : $f(x) = \dfrac{x}{1+|x|}$

$$f(x) = \frac{x}{1-x}, \quad x < 0$$
$$= 0, \quad x = 0$$
$$= \frac{x}{1+x}, \quad x > 0$$

$$\Rightarrow f'(x) = \frac{(1-x)1 - x(-1)}{(1-x)^2}, \quad x < 0$$
$$= 0 \quad , \quad x = 0$$
$$= \frac{(1+x)\cdot 1 - x \cdot 1}{(1+x)^2}, \quad x > 0$$

$f'(0) = 1$ from both RHD and LHD. As $f'(x)$ exists at origin, $f(x)$ is differentiable at origin.

Thus statement-1 is false. Statement-2 is true, $f(x) = |x|$ and so $1 + |x|$ is not differentiable at origin.

25. (a) : Let $f(x) = (a+1)x^2 - 3ax + 4a$ and let α, β be the roots of the equation $f(x) = 0$. The equation will have roots greater than 1 iff

(i) disc ≥ 0 (ii) $\alpha + \beta > 2$

(iii) $(a+1) f(1) > 0$

Now, Disc $\geq 0 \Rightarrow 9a^2 - 16a(a+1) \geq 0$

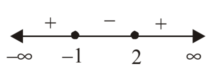
$\Rightarrow -7a^2 - 16a \geq 0 \Rightarrow a(7a + 16) \leq 0$

$\Rightarrow -\dfrac{16}{7} \leq a \leq 0$ ... (i)

$\alpha + \beta > 2 \Rightarrow \dfrac{3a}{a+1} > 2$

$\Rightarrow \dfrac{3a}{a+1} - 2 > 0 \Rightarrow \dfrac{3a - 2a - 2}{a+1} > 0 \Rightarrow \dfrac{a-2}{a+1} > 0$

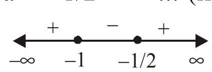
$\Rightarrow a < -1$ or $a > 2$ ... (ii)



and $(a+1)f(1) > 0$

$\Rightarrow (a+1)(a + 1 - 3a + 4a) > 0$

$\Rightarrow a < -1$ or $a > -1/2$ ... (iii)



From (i), (ii) and (iii), we get

$\dfrac{-16}{7} \leq a < -1$ *i.e.* $a \in [-16/7, -1]$.

26. (c) : Let $f(x) = 2x^2 + ax + a^2 - 5$ and let α, β be the roots of $f(x) = 0$. Then both the roots of $f(x) = 0$ will be less than one iff

(i) Disc ≥ 0 (ii) $\alpha + \beta < 2$ (iii) $2f(1) > 0$

Now, Disc $\geq 0 \Rightarrow a^2 - 8(a^2 - 5) \geq 0$

$\Rightarrow 7a^2 - 40 \leq 0$

$\Rightarrow (\sqrt{7}a - \sqrt{40})(\sqrt{7}a + \sqrt{40}) \leq 0$

$\Rightarrow a \in \left[-\sqrt{\dfrac{40}{7}}, \sqrt{\dfrac{40}{7}}\right]$ ... (i)

$\alpha + \beta < 2 \Rightarrow -\dfrac{a}{2} < 2 \Rightarrow a > -4$ ... (ii)

and $2f(1) > 0 \Rightarrow 2(2 + a + a^2 - 5) > 0$

$\Rightarrow a \in \left(-\infty, \dfrac{-1-\sqrt{13}}{2}\right) \cup \left(\dfrac{-1+\sqrt{13}}{2}, \infty\right)$ ... (iii)

From (i), (ii) and (iii), we get

$a \in \left(-\sqrt{\dfrac{40}{7}}, \dfrac{-1-\sqrt{13}}{2}\right) \cup \left(\dfrac{\sqrt{13}-1}{2}, \sqrt{\dfrac{40}{7}}\right).$

27. (d) : Let $f(x) = x^2 + 2(a-3)x + 9$. If 6 lies between the roots of $f(x) = 0$, then we must have the following:

(i) Disc > 0

(ii) $f(6) < 0$ ($\because$ coeff. of x^2 is positive)

Now, Disc $> 0 \Rightarrow 4(a-3)^2 - 36 > 0$

$\Rightarrow (a-3)^2 - 9 > 0 \Rightarrow a^2 - 6a > 0$

$\Rightarrow a(a-6) > 0$

$\Rightarrow a < 0$ or $a > 6$ ... (i)

and $f(6) < 0 \Rightarrow 36 + 12(a-3) + 9 < 0$

$\Rightarrow 12a + 9 < 0 \Rightarrow a < -\dfrac{3}{4}$ ... (ii)

From (i) and (ii), we get

$a < -3/4$ *i.e.* $a \in (-\infty, -3/4)$.

28. (b) : Let $f(x) = (a-3)x^2 - 2ax + 5a$.

For the roots of $f(x) = 0$ to be positive, we must have

(i) Disc ≥ 0 (ii) sum of the roots > 0, and

(iii) $(a-3)f(0) > 0$

Now, Disc $\geq 0 \Rightarrow 4a^2 - 20a(a-3) > 0$

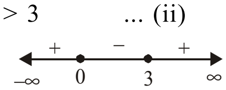
$\Rightarrow -16a^2 + 60a \geq 0 \Rightarrow 4a(4a - 15) \leq 0$

$\Rightarrow 0 \leq a \leq 15/4$ ... (i)

Sum of the roots > 0

$\Rightarrow \dfrac{2a}{a-3} > 0 \Rightarrow \dfrac{a}{a-3} > 0$

$\Rightarrow a < 0$ or $a > 3$ ... (ii)



and $(a-3)f(0) > 0 \Rightarrow (a-3)5a > 0$

$\Rightarrow a(a-3) > 0$

$\Rightarrow a < 0$ or $a > 3$ ... (iii)

From (i), (ii) and (iii), we get

$3 < a \leq 15/4$ *i.e.* $a \in [3, 15/4]$.

29. (a) : We have

$f(x) = \sin^2 x + \sin^2(x + \pi/3) + \cos x \cos(x + \pi/3)$

$= \dfrac{1 - \cos 2x}{2} + \dfrac{1 - \cos(2x + 2\pi/3)}{2} + \dfrac{1}{2}\{2\cos x \cos(x + \pi/3)\}$

$= \dfrac{1}{2}[1 - \cos 2x + 1 - \cos(2x + 2\pi/3) + \cos(2x + \pi/3) + \cos \pi/3]$

$= \dfrac{1}{2}\left[\dfrac{5}{2} - \left\{\cos 2x + \cos\left(2x + \dfrac{2\pi}{3}\right)\right\} + \cos\left(2x + \dfrac{\pi}{3}\right)\right]$

$= \dfrac{1}{2}\left[\dfrac{5}{2} - 2\cos\left(2x + \dfrac{\pi}{3}\right)\cos\dfrac{\pi}{3} + \cos\left(2x + \dfrac{\pi}{3}\right)\right] = \dfrac{5}{4}$ for all x.

$\therefore$ $gof(x) = g(f(x)) = g(5/4) = 1$ [$\because g(5/4) = 1$ (given)]

Hence, $gof(x) = 1$ for all x.

30. (b) : We have

$$f(x)=\log\left(\frac{1+x}{1-x}\right) \text{ and } g(x)=\frac{3x+x^3}{1+3x^2}$$

Therefore, $fog\ (x) = f\ (g(x)) = f\left(\frac{3x+x^3}{1+3x^2}\right)$

$$=\log\left(\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}\right)=\log\left(\frac{(1+x)^3}{(1-x)^3}\right)$$

$$=\log\left(\frac{1+x}{1-x}\right)^3=3\log\left(\frac{1+x}{1-x}\right)=3f(x).$$

31. (a) : $f(x)=\dfrac{ax+b}{cx+d}$.

$\therefore\ fof\ (x) = x \Leftrightarrow f\,(f\,(x)) = x$

$$\Leftrightarrow f\left(\frac{ax+b}{cx+d}\right)=x \Leftrightarrow \frac{a\left(\frac{ax+b}{cx+d}\right)+b}{c\left(\frac{ax+b}{cx+d}\right)+d}=x$$

$$\Leftrightarrow \frac{x(a^2+bc)+ab+bd}{x(a+cd)+bc+d^2}=x$$

Clearly, $d = -a$ satisfies this relation.

32. (a) : $x^2 + y^2 = a^2$ ... (i)

$y = mx + c$ ... (ii)

Substituting the values of y from (ii) in (i), we get

$x^2 + (mx + c)^2 = a^2$

$\Rightarrow\ x^2(1 + m^2) + 2mcx + (c^2 - a^2) = 0$ ... (iii)

When points of intersection are real and distinct, the equation (iii) has two distinct roots.

$\Rightarrow$ Discriminant > 0

$\Rightarrow\ 4m^2c^2 - 4(1 + m^2)(c^2 - a^2) > 0$

$\Rightarrow\ 4a^2(1 + m^2) - 4c^2 > 0$

$\Rightarrow\ a^2(1 + m^2) > c^2 \Rightarrow a^2>\dfrac{c^2}{1+m^2}$

$\Rightarrow\ a>\left|\dfrac{c}{\sqrt{1+m^2}}\right|$

$\Rightarrow\ a >$ (length of the perpendicular from the centre (0, 0) to $y = mx + c$)

Thus, a line intersects a given circle at two distinct points if the length of the perpendicular from the centre is less than the radius of the circle.

33. (d) : When the points of intersection are coincident, the equation (iii) has two equal roots. $\therefore$ Discriminent = 0

$\Rightarrow\ 4m^2c^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$\Rightarrow\ c^2 = a^2(1 + m^2) \Rightarrow a=\left|\dfrac{c}{\sqrt{1+m^2}}\right|$

$\Rightarrow\ a$ = length of the perpendicular from the centre (0, 0) to $y = mx + c$.

Thus, a line touches a circle if the length of the perpendicular from the centre is equal to the radius of the circle.

34. (c) : When the points of intersection are imaginary, the equation (iii) has imaginary roots.

$\therefore$ Discriminant < 0

$\Rightarrow\ 4m^2c^2 - 4(1 + m^2)(c^2 - a^2) < 0$

$\Rightarrow\ a^2(1 + m^2) - c^2 < 0$

$\Rightarrow\ a^2(1 + m^2) < c^2 \quad \Rightarrow\ a<\left|\dfrac{c}{\sqrt{1+m^2}}\right|$

$\Rightarrow\ a <$ length of the perpendicular from the centre (0, 0) to $y = mx + c$.

Thus, a line does not intersect a circle if the length of the perpendicular from the centre is greater than the radius of the circle.

35. (b) : We have $x \ge 1$, $y \ge 2$, $z \ge 3$ and $t \ge 0$, where x, y, z, t are integers.

Let $u = x - 1$, $v = y - 2$, $w = z - 3$, then

$x \ge 1 \ \Rightarrow\ u \ge 0 \qquad y \ge 2 \ \Rightarrow\ v \ge 0$

$z \ge 3 \ \Rightarrow\ w \ge 0$

Thus, we have

$u + 1 + v + 2 + w + 3 + t = 29$

where $u \ge 0$, $v \ge 0$, $w \ge 0$, $t \ge 0$

$u + v + w + t = 23$

The total number of solutions of this equation is

${}^{23+4-1}C_{4-1} = {}^{26}C_3 = 2600.$

36. (d) : Let $x_4 = k$, then $x_1 + x_2 + x_3 + 4x_4 = 20$

$\Rightarrow\ x_1 + x_2 + x_3 = 20 - 4k$ ... (i)

Since x_1, x_2, x_3 and x_4 are non-negative integers. Therefore, $20 \ge 20 - 4k \ge 0 \Rightarrow\ 0 \le k \le 5$. For a given value of k, the total number of integral solutions of (i) is

${}^{20-4k+3-1}C_{3-1} = {}^{22-4k}C_2$

But k varies from 0 to 5. So, the total number of integral solutions of the given equation is

$$\sum_{k=0}^{5} {}^{22-4k}C_2=\sum_{k=0}^{5}(8k^2-86k+231)$$

$$=8\sum_{k=0}^{5}k^2-86\sum_{k=0}^{5}k+231\times 6$$

$= 8 \times 55 - 86 \times 15 + 231 \times 6 = 536.$

37. (A) → (q); (B) → (r); (C) → (p); (D) → (q)

(A) LHL of the function $\dfrac{f(x)}{x}$

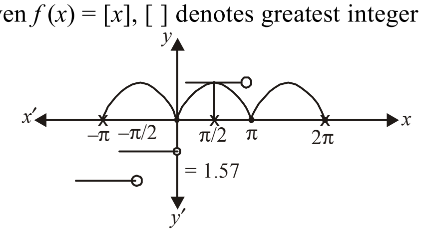
$$=\underset{x\to 0^-}{\text{Lt}}\frac{f(x)}{x}=\underset{x\to 0^-}{\text{Lt}}\frac{|\sin x|}{x}=\underset{x\to 0^-}{\text{Lt}}\frac{\sin x}{-x}=-1$$

$$\text{RHL}=\underset{x\to 0^+}{\text{Lt}}\frac{|\sin x|}{x}=\underset{x\to 0^+}{\text{Lt}}\frac{\sin x}{x}=1$$

as LHL at $x = 0 \ne$ RHL at $x = 0$

$\Rightarrow$ limit of $f\,(x)$ at $x = 0$ does not exist.

(B) Given $f\,(x) = [x]$, [ ] denotes greatest integer function.



From the graph we note that the curve of $y = [x]$ and if $y = f(x)$ intersect at two points $x = 0$ and $x = \dfrac{\pi}{2}$.

$\therefore$ Total number of solutions obtained are 2.

(C) $f(x) = |\sin x|$ and $f(x) = 5^x + 5^{-x}$

using fact $0 \le |\sin x| \le 1$

$\therefore -1 \le \sin x \le 1$ and $5^x + 5^{-x} \ge 2$

(using AM $\ge$ GM) as $f(x) = 5^x + 5^{-x}$ given

$\Rightarrow$ LHS having maximum value of $f(x)$ is 1 but RHS having minimum value 2.

Hence no solution exists for the given equation.

(D) If $|\sin x| = \sin x$, $x > 0$

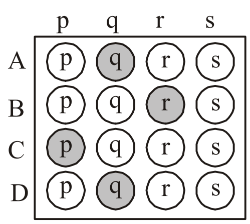
$$\left(\frac{d}{dx}\sin x\right)_{x>0} = (\cos x)_0 = 1$$

If $|\sin x| = -\sin x$, $x < 0$

$$\left(\frac{d}{dx}(-\sin x)\right)_{x<0} = (-\cos x)_0 = -1$$

$\therefore$ L.H.D. $\ne$ R.H.D

$\therefore$ $\left(\dfrac{d}{dx}|\sin x|\right)_{x=0}$ does not exist.



38. (A) $\rightarrow$ (s); (B) $\rightarrow$ (r); (C) $\rightarrow$ (q); (D) $\rightarrow$ (p)

(A) $1 + f(x) = 1 + \text{sgn}(x) = \begin{cases} 1+1 = 2, & \text{if } x > 0 \\ 1+0 = 1, & \text{if } x = 0 \\ 1-1 = 0, & \text{if } x < 0 \end{cases}$

$\therefore$ range is set of number $\{2, 1, 0\}$.

(B) $\sqrt{1+f(x)} = \sqrt{1+\text{sgn}(x)}$ is defined $\forall\ x \in R$

but sgn (x) is discontinuous at $x = 0$

$\therefore \sqrt{1+\text{sgn}(x)}$ is also discontinuous at $x = 0$

more over $\underset{x\to 0^-}{\text{LHL}} \ne \underset{x\to 0^+}{\text{RHL}}$ i.e. non removable discontinuity

$\therefore \sqrt{1+f(x)}$ is discontinuos at $x = 0$

(C) $f(x) = \text{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$

$\therefore\ f(x) - 1 = \begin{cases} 0 & \text{if } x > 0 \\ -1 & \text{if } x = 0 \\ -2 & \text{if } x < 0 \end{cases}$

$\therefore$ range of sgn $x - 1$ is $\{0, -1, -2\}$

(D) To find RHL at $x = 0$, putting $x = 0 + h$ (where h is small number > 0) and $h \to 0$

$$\text{RHL} = \underset{x\to 0^+}{\text{Lt}}\ g(x) = \underset{h\to 0}{\text{Lt}}\ g(0+h) = \underset{h\to 0}{\text{Lt}}\ |\text{sgn}(h)| + 2 = 3$$

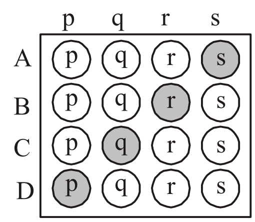
Similarly, $\text{LHL} = \underset{x\to 0^-}{\text{Lt}}\ g(x) = \underset{h\to 0}{\text{Lt}}\ g(0-h)$

$$= \underset{h\to 0}{\text{Lt}}\ |\text{sgn}\,(-h)| + 2 = 3$$

and $g(0) = |\text{sgn} \times 0| + 2 = 2$

As LHL = RHL = $3 \ne g(0) = 2$

$\therefore\ g(x) = 2 + |f(x)|$ has removable discontinuity at $x = 0$.



39. (A) $\rightarrow$ (r); (B) $\rightarrow$ (p); (C) $\rightarrow$ (s); (D) $\rightarrow$ (q)

(A) Consider

$(1 + x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots + C_nx^n$ ...(i)

Multiplying by x to both sides of (i) we get

$x(1 + x)^n = C_0x + C_1x^2 + C_2x^3 + C_3x^4 + \ldots + C_nx^{n+1}$ ... (ii)

Now differentiating (ii) w.r.t to x both sides we get

$nx(1+x)^{n-1} + (1 + x)^n = C_0 + 2C_1x + 3C_2x^2 + 4C_3x^3 + \ldots + (n + 1)C_nx^{n+1}$ ...(iii)

Now putting $x = 1$ in equation (iii) we get

$2^{n-1}(n + 2) = C_0 + 2C_1 + 3C_2 + \ldots + (n + 1)C_n$.

(B) Putting $x = -1$ to both sides of above equation (iii) we get

$C_0 - 2C_1 + 3C_2 - 4C_3 + \ldots + (-1)^n(n + 1)C_n = 0$.

(C) Let $S = C_0 + (C_0 + C_1) + (C_0 + C_1 + C_2) + \ldots + (C_0 + C_1 + C_2 \ldots + C_{n-1})$

$S = nC_0 + (n - 1)C_1 + (n - 2)C_2 + \ldots + 1C_{n-1} + 0C_n$ ...(i)

$\therefore$ Writing (i) in reversed order and using

$C_0, C_1, \ldots, C_n$ instead of $C_n, C_{n-1}, C_{n-2}, \ldots, C_0$

$S = 0C_0 + 1C_1 + 2C_2 + \ldots + (n - 1)C_{n-1} + nC_n$ ...(ii)

Now adding (i) and (ii) we get

$2S = n[C_0 + C_1 + C_2 + \ldots + C_n]$ $\therefore S = n2^{n-1}$.

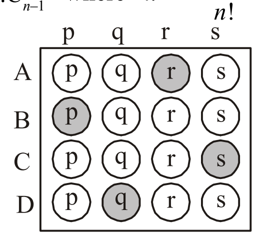
(D) Consider $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3) \ldots (C_{n-1} + C_n)$

$$= C_0C_1C_2 \ldots C_{n-1}\left[\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\left(1+\frac{C_3}{C_2}\right)\ldots\left(1+\frac{C_n}{C_{n-1}}\right)\right]$$

$$= C_0C_1C_2 \ldots C_{n-1}\left[\left(\frac{n+1}{1}\right)\left(\frac{n+1}{2}\right)\left(\frac{n+1}{3}\right)\ldots\left(\frac{n+1}{n}\right)\right]$$

$$= C_0C_1C_2 \ldots C_{n-1}\frac{(n+1)^n}{n!}$$

$= k\,C_0C_1C_2 \ldots C_{n-1}$ where $k = \dfrac{(n+1)^n}{n!}$



40. (A) → (s); (B) → (p); (C) → (q); (D) → (r)

$$y = 1 + \frac{dy}{dx} + \frac{\left(\frac{dy}{dx}\right)^2}{2!} + \frac{\left(\frac{dy}{dx}\right)^3}{3!} + \text{.........}\infty$$

$$y = e^{\frac{dy}{dx}} \Rightarrow \frac{dy}{dx} = \log y \Rightarrow \text{degree } 1$$

Again $\frac{dy}{dx} + \frac{1}{3}\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = 0$

Highest order is 2 whose exponent is also 2.
so order is 2 and degree is 2

Also $x = 1 + xy\frac{dy}{dx} + \frac{(xy)^2}{2!}\left(\frac{dy}{dx}\right)^2 + ...$

$$\Rightarrow x = e^{xy\frac{dy}{dx}} \Rightarrow xy\frac{dy}{dx} = \log x \Rightarrow y\frac{dy}{dx} = \frac{\log x}{x}$$

$\Rightarrow$ order is 1 and degree is 1.

and $\left(\frac{d^2y}{dx^2}\right)^4 = y + \left(\frac{dy}{dx}\right)^3 \Rightarrow$ order 2 and degree 4.

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

41. (9) : This is a case equivalent to distributing 12 identical into 3 identical boxes is same as $x + y + z = 12$ taking only different combinations of x, y and z.

Case 1 : $x = y = z = 4$ only 1 way

Case 2 : $x = y + z$, $2x + z = 12$, $x \neq z$

(a) $x > z$, $x - z = a$, $a \geq 1 \Rightarrow x = z + a$

So, $2x + z = 12 \Rightarrow 3z + 2a = 12$, $a \geq 1$, $z \geq 0$

i.e., Coefficient of x^{12} in $(1 + x^3 + x^6 + ...)(x^2 + x^4 + ...) = 2$.

(b) $x < z$, $z - x = a$, $z = x + q$

$3x + a = 12$, $a \geq 1$, $x \geq 0$

Coefficient of x^{12} in

$(1 + x^3 + x^6 + x^9 + ...)(x + x^2 + ...) = 4$

Case 3 : $x + y + z = 12$, $x \neq y \neq z$

Taking only different combinations of x, y and z.

Let $x > y > z \Rightarrow y - z = a \Rightarrow y = z + a$, $a \geq 1$

$x - y = b \Rightarrow x = z + a + b$, $b \geq 1$

$x + y + z = 12 \Rightarrow 3z + 2a + b = 12$

$a, b \geq 1$, $z > 0$

Coefficient of x^{12} in $(1 + x^3 + x^6 + ...)$

$(x^2 + x^4 + x^6 + ...)(x + x^2 + x^3 + ...) = 12$

$\therefore$ Total number of ways $= 1 + 2 + 4 + 12 = 19$

42. (1) : $x_1 + x_2 + x_1x_2 = a$ ......(i)

$x_1x_2 + x_1^2x_2 + x_1x_2^2 = b$ ......(ii)

$x_1^2x_2^2 = c$ ......(iii)

From Eq. (ii), $x_1x_2(1 + x_1 + x_2) = b$

$\Rightarrow x_1x_2(1 + a - x_1x_2) = b \Rightarrow x_1x_2(1 + a) - c = b$

$$\Rightarrow x_1x_2 = \frac{b+c}{a+1} \Rightarrow x_1x_2\left(\frac{a+1}{b+c}\right) = 1$$

43. (4) : $f(3) = 3f(1) = 3$, $f(4) = f(2 + 1 + 1) = 2 + 1 + 1 = 4$ and so on. In general, we get $f(r) = r$ for $r \in N$

$$\Rightarrow \lim_{n\to\infty}\frac{\sum_{r=1}^{n}(4r)f(3r)}{n^3} = \lim_{n\to\infty}\frac{12n(n+1)(2n+1)}{6n^3} = 4$$

44. (1) : For $c < 1$; $\int_c^1 (8x^2 - x^5)dx = \frac{16}{3}$

$$\Rightarrow \frac{8}{3} - \frac{1}{6} - \frac{8c^3}{3} + \frac{c^6}{6} = \frac{16}{3}$$

$$\Rightarrow c^3\left[-\frac{8}{3} + \frac{c^3}{6}\right] = \frac{16}{3} - \frac{8}{3} + \frac{1}{6} = \frac{17}{6}$$

$\Rightarrow c = -1$

Again, for $c \geq 1$, none of the values of c satisfy the required condition that $\int_1^c (8x^2 - x^5)dx = \frac{16}{3}$

$\therefore c + 2 = 1$

45. (5) : $D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & -4 & -2 \\ 2 & 5 & -2 \end{vmatrix} = 45$, $D_{1/x} = \begin{vmatrix} 1 & 2 & 1 \\ 2 & -4 & -2 \\ 3 & 5 & -2 \end{vmatrix} = 36$,

$$D_{1/y} = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & -2 \\ 2 & 3 & -2 \end{vmatrix} = 9, \quad D_{1/z} = \begin{vmatrix} 1 & 2 & 1 \\ 3 & -4 & 2 \\ 2 & 5 & 3 \end{vmatrix} = -9$$

$x = \frac{5}{4}$, $y = 5$, $z = -5$

Hence, value of y is 5.

41	42	43	44	45
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

• END •