SOLUTIONS

1. The highest frequency is 75 corresponds to class 20-25. So, the modal class is 20-25.

2. Given, $\frac{a_{18}}{a_{11}} = \frac{3}{2} \Rightarrow \frac{a+17d}{a+10d} = \frac{3}{2}$

$\Rightarrow 2a + 34d = 3a + 30d \Rightarrow a = 4d$ ... (i)

Now, $\frac{a_{21}}{a_5} = \frac{a+20d}{a+4d} = \frac{4d+20d}{4d+4d}$ [Using (i)]

$= \frac{24d}{8d} = \frac{3}{1}$

$\therefore$ Required ratio = 3 : 1

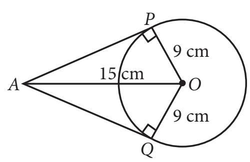
OR

All two digit numbers are 10, 11,....., 99.
Here, $a = 10$, $d = 1$, $n = 90$

$\therefore$ Required sum, $S_n = \frac{n}{2}(10+99) = \frac{90}{2}(109)$

$= 45 \times 109 = 4905$

3. Since, tangents drawn from an external point of a circle are equal.



$\therefore AP = AQ$
Also, $OP \perp AP$ and $OQ \perp AQ$
[$\because$ Tangent at any point of a circle is perpendicular to the radius through the point of contact.]
$\therefore$ In ΔAOP,
$AP^2 = AO^2 - OP^2$ [By Pythagoras theorem]
$= 15^2 - 9^2 = 225 - 81 = 144$
$\Rightarrow AP = 12$ cm

4. We have, $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$
$\Rightarrow 12abx^2 - 9a^2x + 8b^2x - 6ab = 0$
$\Rightarrow 3ax(4bx - 3a) + 2b(4bx - 3a) = 0$
$\Rightarrow (4bx - 3a)(3ax + 2b) = 0$
$\Rightarrow 4bx - 3a = 0$ or $3ax + 2b = 0$

$\Rightarrow x = \frac{3a}{4b}$ or $x = \frac{-2b}{3a}$

5. Let the rainfall be x.
Now, volume of water on roof = volume of cone

$\Rightarrow 44 \times 10 \times x = \frac{1}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times 7$

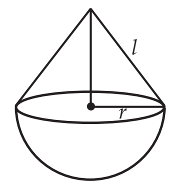
$\Rightarrow x = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{1}{44} \times \frac{1}{10}$

$\Rightarrow x = \frac{1}{240}$ m $= \frac{1}{240} \times 100$ cm $= \frac{5}{12}$ cm

Hence, required rainfall is 5/12 cm.

OR

Let r be the radius of hemisphere and conical part. Also, let l be the slant height of conical part.



Given, Surface area of hemisphere
= Surface area of conical part
$\Rightarrow 2\pi r^2 = \pi rl \Rightarrow 2r = l$

$\Rightarrow \frac{r}{l} = \frac{1}{2}$

$\therefore$ Required ratio = 1 : 2

6. We know, the empirical relationship is
Mode = 3 Median – 2 Mean
= 3(9.6) – 2(10.5) [$\because$ Median = 9.6 and Mean = 10.5]
= 28.8 – 21.0 = 7.8

7. Let the two consecutive positive integers be x and $x + 1$.
According to question, $x^2 + (x + 1)^2 = 61$
$\Rightarrow x^2 + x^2 + 2x + 1 = 61$
$\Rightarrow 2x^2 + 2x = 60 \Rightarrow x^2 + x = 30$

Adding $\left(\frac{1}{2}\right)^2$ on both sides, we get

$$x^2 + x + \frac{1}{4} = 30 + \frac{1}{4}$$

$\Rightarrow \left(x + \frac{1}{2}\right)^2 = \frac{121}{4} \Rightarrow x + \frac{1}{2} = \pm\frac{11}{2} \Rightarrow x = \pm\frac{11}{2} - \frac{1}{2}$

$\Rightarrow x = \frac{11}{2} - \frac{1}{2}$ or $x = -\frac{11}{2} - \frac{1}{2}$

$\Rightarrow x = 5$ or $x = -6$
$\Rightarrow x = 5$ [Since x is a positive integer]
And $x + 1 = 6$
$\therefore$ The two consecutive positive integers are 5 and 6.

8. Since p, q, r are in A.P.
$\therefore q - p = r - q \Rightarrow 2q = p + r \Rightarrow p + r - 2q = 0$
$\Rightarrow p^3 + r^3 + (-2q)^3 = 3 \times p \times r \times (-2q)$
[$\because$ If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$]
$\Rightarrow p^3 + r^3 - 8q^3 = -6pqr$

OR

The given A.P. is 4, 7, 10, 13, …

Here, $a = 4$, $d = 7 - 4 = 3$

Let the n^{th} term of the A.P. be 49.

Then, $a_n = a + (n - 1)d \Rightarrow 49 = 4 + (n - 1)(3)$

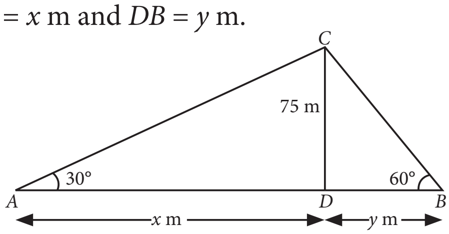
$\Rightarrow\ 45 = 3(n - 1) \Rightarrow n - 1 = 15\ \Rightarrow\ n = 16$

Hence, 16^{th} term of the A.P. is 49.

9. Let $CD = 75$ m be the height of the building. Let A and B be the points of observations such that the angle of elevation at A is 30° and the angle of elevation at B is 60°.

$\therefore\ \angle CAD = 30°$ and $\angle CBD = 60°$

Let $AD = x$ m and $DB = y$ m.



In right angled ΔADC,

$$\tan 30° = \frac{CD}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{x} \Rightarrow x = 75\sqrt{3}\text{ m} \quad \text{…(i)}$$

In right angled ΔBDC,

$$\tan 60° = \frac{CD}{DB} \Rightarrow \sqrt{3} = \frac{75}{y} \Rightarrow y = \frac{75}{\sqrt{3}}\text{ m} \quad \text{…(ii)}$$

The distance between two men is AB,

i.e., $AB = AD + DB = x + y$

$$\Rightarrow AB = \left(75\sqrt{3} + \frac{75}{\sqrt{3}}\right) \quad \text{[Using (i) and (ii)]}$$

$$\Rightarrow AB = \left(\frac{225+75}{\sqrt{3}}\right) = \frac{300}{\sqrt{3}} = \frac{300\sqrt{3}}{3}$$

$= 100\sqrt{3} = 100 \times 1.73 \Rightarrow AB = 173$ m

10. Join OP and OS.

Since, length of tangents drawn from an external point to a circle are equal.

$\therefore\ AP = AS$ [Tangents from A] ...(i)

$CQ = CR$ [Tangents from C] ...(ii)

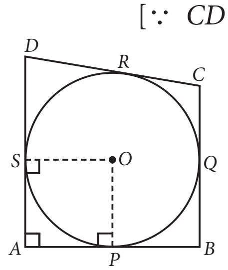
$DR = DS$ [Tangents from D] ...(iii)

Now, $CQ = CR \Rightarrow CR = 18$ cm

[$\because\ CQ = 18$ cm (given)]

$DR = DC - CR = 35 - 18 = 17$ cm

[$\because\ CD = 35$ cm (given)]



$\therefore\ DS = 17$ cm [Using (iii)]

$AS = AD - DS = 40 - 17 = 23$ cm

[$\because\ AD = 40$ cm (given)]

$\therefore\ AP = 23$ cm [Using (i)]

Now, $OP \perp AP$ and $OS \perp AS$

[$\because$ Tangent at any point of circle is perpendicular to the radius through the point of contact]

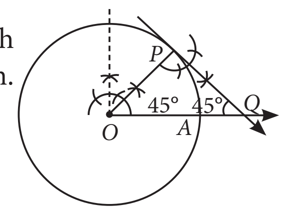
Also, $\angle DAB = 90°$ [Given]

Since, all angles are of 90° and adjacent sides are equal in $APOS$, so $APOS$ is a square.

$\therefore\ OP = OS = AS = AP = 23$ cm

Thus, radius of the circle is 23 cm.

11. Steps of construction :



Step-I : Draw a circle with centre O and radius, $OP = 7$ cm.

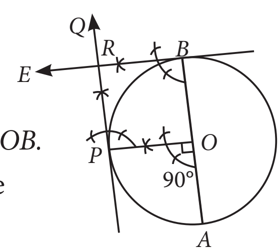
Step-II : Construct an angle AOP equal to complement of 45° *i.e.*, $\angle AOP = 45°$.

Step-III : Draw perpendicular to OP at P which meets OA produced at Q.

$\therefore\ PQ$ is the required tangent such that $\angle OQP = 45°$.

OR

Steps of construction :



Step-I : Draw a circle with centre O and radius 3 cm.

Step-II : Draw any diameter AOB.

Step-III : Take a point P on the circle such that $\angle AOP = 90°$.

Step-IV : Draw $PQ \perp OP$ and $BE \perp OB$. Let PQ and BE intersect at R.

Hence, RB and RP are the required tangents inclined at an angle of 90°.

12. Here we have, the cumulative frequency distribution more than type. So, first we convert it into an ordinary frequency distribution. We observe that there are 80 students getting marks greater than or equal to 0 and 77 students have secured greater than or equal to 10. Therefore, the number of students getting marks between 0 and 10 is 80 – 77 = 3.

Similarly, the number of students getting marks between 10 and 20 is 77 – 72 = 5 and so on.

Thus, we obtain the following frequency distribution.

Marks	Frequency (f_i)	Class Mark (x_i)	f_ix_i
0-10	80 – 77 = 3	5	15
10-20	77 – 72 = 5	15	75

20-30	72 – 65 = 7	25	175
30-40	65 – 55 =10	35	350
40-50	55 – 43 = 12	45	540
50-60	43 – 28 = 15	55	825
60-70	28 – 16 = 12	65	780
70-80	16 – 10 = 6	75	450
80-90	10 – 8 = 2	85	170
90-100	8 – 0 = 8	95	760
Total	$\sum f_i = 80$		$\sum f_i x_i = 4140$

$\therefore$ Mean, $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{4140}{80} = 51.75$

Hence, mean marks scored by the students is 51.75.

13. (i) We have, $r = 14$ cm, $h = 25$ cm

Volume of juice in the jar $= \pi r^2 h$

$$= \frac{22}{7} \times (14)^2 \times 25 = 15400 \text{ cu. cm}$$

(ii) Side of ice cube = 5.6 cm

$\therefore$ Volume of each ice cube $= (5.6)^3 = 175.616$ cu. cm

14. (i) In ΔPAB, $\tan 45° = \dfrac{AB}{PA} \Rightarrow 1 = \dfrac{20}{PA}$

$\Rightarrow PA = 20$ m

So, required distance between foot of building and P is 20 m.

(ii) Let h be the height of antenna from the top of the building.

Then, in ΔPAD, $\tan 60° = \dfrac{AD}{PA}$

$\Rightarrow \sqrt{3} = \dfrac{AB + BD}{PA}$

$\Rightarrow \sqrt{3} \times 20 = 20 + h$

$\Rightarrow h = 20(\sqrt{3} - 1) = 20\,(1.732 - 1)$

$= 20 \times 0.732 = 14.64$ m

$\therefore$ Required height $= AD = AB + BD$

$= 20 + 14.64 = 34.64$ m

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Self Evaluation Sheet

Once you complete ***SQP-3****, check your answers with the given solutions and fill your marks in the marks obtained column according to the marking scheme. Performance Analysis Table given at the bottom will help you to check your readiness.*

Q. No.	Chapter	Marks Per Question	Marks Obtained
1	Statistics	2	
2	Arithmetic Progressions / Arithmetic Progressions	2	
3	Circles	2	
4	Quadratic Equations	2	
5	Surface Areas and Volumes / Surface Areas and Volumes	2	
6	Statistics	2	
7	Quadratic Equations	3	
8	Arithmetic Progressions / Arithmetic Progressions	3	
9	Some Applications of Trigonometry	3	
10	Circles	3	
11	Constructions / Constructions	4	
12	Statistics	4	
13	Surface Areas and Volumes	2×2	
14	Some Applications of Trigonometry	2×2	
	Total Marks	**40**	....
		Percentage	.............%

Performance Analysis Table

If your marks is

> 90%	**TREMENDOUS!**	➢ *You are done! Keep on revising to maintain the position.*
81-90%	**EXCELLENT!**	➢ *You have to take only one more step to reach the top of the ladder. Practise more.*
71-80%	**VERY GOOD!**	➢ *A little bit of more effort is required to reach the 'Excellent' bench mark.*
61-70%	**GOOD!**	➢ *Revise thoroughly and strengthen your concepts.*
51-60%	**FAIR PERFORMANCE!**	➢ *Need to work hard to get through this stage.*
40-50%	**AVERAGE!**	➢ *Try hard to boost your average score.*