Marking Scheme

Mathematics Class X (2017-18)

Section A

S.No.	Answer	Marks
1.	Non terminating repeating decimal expansion.	[1]
2.	$k = \pm 4$	[1]
3.	$a_{11} = -25$	[1]
4.	(0, 5)	[1]
5.	9 : 49	[1]
6.	25	[1]

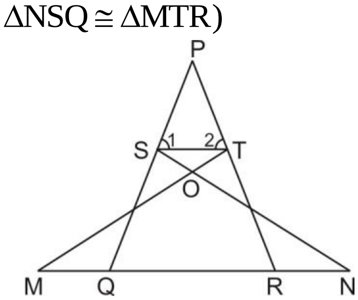
Section B

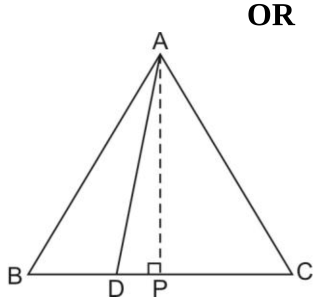
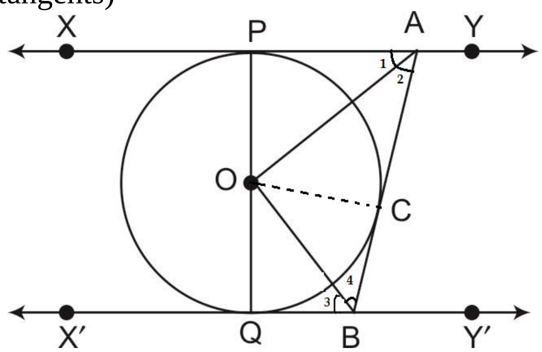
7.	LCM (p, q) = a^3b^3 HCF (p, q) = a^2b LCM (p, q) × HCF (p, q) = $a^5b^4 = (a^2b^3)(a^3 b) = pq$	[1/2] [1/2] [1]
8.	$S_n = 2n^2 + 3n$ $S_1 = 5 = a_1$ $S_2 = a_1 + a_2 = 14 \Rightarrow a_2 = 9$ $d = a_2 - a_1 = 4$ $a_{16} = a_1 + 15d = 5 + 15(4) = 65$	[1/2] [1/2] [1/2] [1/2]
9.	For pair of equations $kx + 1y = k^2$ and $1x + ky = 1$ We have: $\frac{a_1}{a_2} = \frac{k}{1}, \frac{b_1}{b_2} = \frac{1}{k}, \frac{c_1}{c_2} = \frac{k^2}{1}$ For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ $\therefore \frac{k}{1} = \frac{1}{k} \Rightarrow k^2 = 1 \Rightarrow k = 1, -1$ ...(i) and $\frac{1}{k} = \frac{k^2}{1} \Rightarrow k^3 = 1 \Rightarrow k = 1$ ...(ii) From (i) and (ii), k = 1	 [1/2] [1/2] [1/2] [1/2]
10.	Since $\left(1, \frac{p}{3}\right)$ is the mid-point of the line segment joining the points (2, 0) and $\left(0, \frac{2}{9}\right)$ therefore, $\frac{p}{3} = \frac{0 + \frac{2}{9}}{2} \Rightarrow p = \frac{1}{3}$ The line $5x + 3y + 2 = 0$ passes through the point (–1, 1) as 5(–1) + 3(1) + 2 = 0	[1] [1]
11.	(i) P(square number) = $\frac{8}{113}$ (ii) P(multiple of 7) = $\frac{16}{113}$	[1] [1]

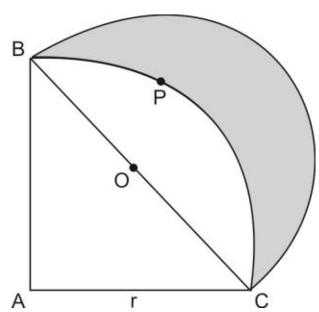
12.	Let number of red balls be = x $\therefore$ P(red ball) = $\frac{x}{12}$ **If 6 more red balls are added:** The number of red balls = x + 6 P(red ball) = $\frac{x+6}{18}$ Since, $\frac{x+6}{18} = 2\left(\frac{x}{12}\right) \Rightarrow x = 3$ $\therefore$ There are 3 red balls in the bag.	[1/2] [1] [1/2]

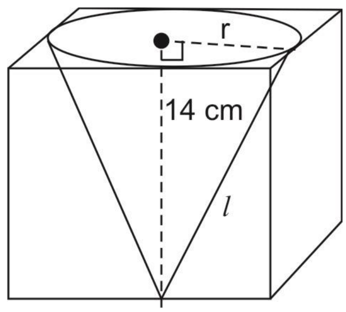
Section C

13.	Let n = 3k, 3k + 1 or 3k + 2. (i) When n = 3k: n is divisible by 3. n + 2 = 3k + 2 $\Rightarrow$ n + 2 is not divisible by 3. n + 4 = 3k + 4 = 3(k + 1) + 1 $\Rightarrow$ n + 4 is not divisible by 3. (ii) When n = 3k + 1: n is not divisible by 3. n + 2 = (3k + 1) + 2 = 3k + 3 = 3(k + 1) $\Rightarrow$ n + 2 is divisible by 3. n + 4 = (3k + 1) + 4 = 3k + 5 = 3(k + 1) + 2 $\Rightarrow$ n + 4 is not divisible by 3. (iii) When n = 3k + 2: n is not divisible by 3. n + 2 = (3k + 2) + 2 = 3k + 4 = 3(k + 1) + 1 $\Rightarrow$ n + 2 is not divisible by 3. n + 4 = (3k + 2) + 4 = 3k + 6 = 3(k + 2) $\Rightarrow$ n + 4 is divisible by 3. Hence exactly one of the numbers n, n + 2 or n + 4 is divisible by 3.	[1] [1] [1]
14.	Since $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are the two zeroes therefore, $\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\frac{1}{3}(3x^2-5)$ is a factor of given polynomial. We divide the given polynomial by $3x^2 - 5$. $3x^2 - 5 \overline{)\,3x^4 + 6x^3 - 2x^2 - 10x - 5\,}(x^2 + 2x + 1$ $\pm 3x^4 \quad \mp 5x^2$ $6x^3 + 3x^2 - 10x - 5$ $\pm 6x^3 \quad \mp 10x$ $3x^2 - 5$ $\pm 3x^2 \mp 5$ 0 For other zeroes, $x^2 + 2x + 1 = 0 \Rightarrow (x + 1)^2 = 0, x = -1, -1$ $\therefore$ Zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and -1.	[1] [1] [1]

15.	Let the ten's and the units digit be y and x respectively. So, the number is 10y + x. The number when digits are reversed is 10x + y. Now, 7(10y + x) = 4(10x + y) $\Rightarrow$ 2y = x …(i) Also x – y = 3 …(ii) Solving (1) and (2), we get y = 3 and x = 6. Hence the number is 36.	 [1/2] [1/2] [1] [1/2] [1/2]
16.	Let x-axis divides the line segment joining (–4, –6) and (–1, 7) at the point P in the ratio 1 : k. Now, coordinates of point of division $P\left(\frac{-1-4k}{k+1}, \frac{7-6k}{k+1}\right)$ Since P lies on x-axis, therefore $\frac{7-6k}{k+1} = 0$ $\Rightarrow$ 7 – 6k = 0 $\Rightarrow$ $k = \frac{7}{6}$ Hence the ratio is $1 : \frac{7}{6} = 6 : 7$ Now, the coordinates of P are $\left(\frac{-34}{13}, 0\right)$. **OR** Let the height of parallelogram taking AB as base be h. Now AB = $\sqrt{(7-4)^2+(2+2)^2} = \sqrt{3^2+4^2} = 5$ units. Area (Δ ABC) = $\frac{1}{2}\left[4(2-9)+7(9+2)+0(-2-2)\right] = \frac{49}{2}$ sq units. Now, $\frac{1}{2} \times AB \times h = \frac{49}{2}$ $\Rightarrow \frac{1}{2} \times 5 \times h = \frac{49}{2}$ $\Rightarrow h = \frac{49}{5} = 9.8$ units.	[1/2] [1] [1/2] [1] [1] [1] [1]
17.	$\angle SQN = \angle TRM$ (CPCT as $\Delta NSQ \cong \Delta MTR$)  Since, $\angle P + \angle 1 + \angle 2 = \angle P + \angle PQR + \angle PRQ$ (Angle sum property) $\Rightarrow \angle 1 + \angle 2 = \angle PQR + \angle PRQ$ $\Rightarrow 2\angle 1 = 2\angle PQR$ (as $\angle 1 = \angle 2$ and $\angle PQR = \angle PRQ$) $\angle 1 = \angle PQR$	[1] [1]

	Also $\angle 2 = \angle PRQ$ And $\angle SPT = \angle QPR$ (common) $\Delta PTS \sim \Delta PRQ$ (By AAA similarity criterion)	[1]
	OR  Construction: Draw $AP \perp BC$ In ΔADP, $AD^2 = AP^2 + DP^2$ $AD^2 = AP^2 + (BP - BD)^2$ $AD^2 = AP^2 + BP^2 + BD^2 - 2(BP)(BD)$ $AD^2 = AB^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right)$ $AD^2 = \frac{7}{9}AB^2 \ (\because BC = AB)$ $9AD^2 = 7AB^2$	[1/2] [1/2] [1/2] [1] [1/2]
18.	Join OC In Δ OPA and Δ OCA OP = OC (radii of same circle) PA = CA (length of two tangents)  AO = AO (Common) $\therefore$ Δ OPA $\cong$ Δ OCA (By SSS congruency criterion) Hence, $\angle 1 = \angle 2$ (CPCT) Similarly $\angle 3 = \angle 4$ Now, $\angle PAB + \angle QBA = 180°$ $\Rightarrow 2\angle 2 + 2\angle 4 = 180°$ $\Rightarrow \angle 2 + \angle 4 = 90°$ $\Rightarrow \angle AOB = 90°$ (Angle sum property)	[1] [1] [1]

19.	$\dfrac{\text{cosec}^2 63° + \tan^2 24°}{\cot^2 66° + \sec^2 27°} + \dfrac{\sin^2 63° + \cos 63° \sin 27° + \sin 27° \sec 63°}{2(\text{cosec}^2 65° - \tan^2 25°)}$	
	$= \dfrac{\text{cosec}^2 63° + \tan^2 24°}{\tan^2(90° - 66°) + \text{cosec}^2(90° - 27°)} + \dfrac{\sin^2 63° + \cos 63° \cos(90° - 27°) + \sin 27° \text{cosec}(90° - 63°)}{2[\text{cosec}^2 65° - \cot^2(90° - 25°)]}$	[1]
	$= \dfrac{\text{cosec}^2 63° + \tan^2 24°}{\tan^2 24° + \text{cosec}^2 63°} + \dfrac{\sin^2 63° + \cos^2 63° + \sin 27° \text{ cosec } 27°}{2(\text{cosec}^2 65° - \cot^2 65°)}$	[1]
	$= 1 + \dfrac{1+1}{2(1)} = 2$	[1]
	OR	
	$\sin\theta + \cos\theta = \sqrt{2}$	
	$\Rightarrow (\sin\theta + \cos\theta)^2 = \left(\sqrt{2}\right)^2$	[1/2]
	$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$	
	$\Rightarrow 1 + 2\sin\theta\cos\theta = 2$	
	$\Rightarrow \sin\theta\cos\theta = \dfrac{1}{2}$ ...(i)	[1]
	we know, $\sin^2\theta + \cos^2\theta = 1$ ....(ii)	[1/2]
	Dividing (ii) by (i) we get	
	$\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{1/2}$	
	$\Rightarrow \tan\theta + \cot\theta = 2$	[1]
20.	We know, AC = r	
	In Δ ACB, $BC^2 = AC^2 + AB^2$	
	$\Rightarrow BC = AC\sqrt{2}$ $(\because AB = AC)$	
	$\Rightarrow BC = r\sqrt{2}$	[1]
		
	Required area = ar(Δ ACB) + ar(semicircle on BC as diameter) –ar(quadrant ABPC)	
	$= \dfrac{1}{2} \times r \times r + \dfrac{1}{2} \times \pi \times \left(\dfrac{r\sqrt{2}}{2}\right)^2 - \dfrac{1}{4}\pi r^2$	
	$= \dfrac{r^2}{2} + \dfrac{\pi r^2}{4} - \dfrac{\pi r^2}{4}$	[1]
	$= \dfrac{r^2}{2} = \dfrac{196}{2} \text{cm}^2 = 98 \text{ cm}^2$	[1]

21.	Let the area that can be irrigated in 30 minute be A m^2. Water flowing in canal in 30 minutes = $\left(10,000\times\frac{1}{2}\right)\text{ m} = 5000\text{ m}$ Volume of water flowing out in 30 minutes = $(5000\times6\times1.5)\text{ m}^3 = 45000\text{ m}^3$ …(i) Volume of water required to irrigate the field = $A\times\frac{8}{100}\text{ m}^3$ …(ii) Equating (i) and (ii), we get $A\times\frac{8}{100} = 45000$ A = 562500 m^2. **OR** $l = \sqrt{7^2+14^2} = 7\sqrt{5}$ Surface area of remaining solid = $6l^2 - \pi r^2 + \pi rl$, where r and l are the radius and slant height of the cone.  $= 6\times14\times14-\frac{22}{7}\times7\times7+\frac{22}{7}\times7\times7\sqrt{5}$ $= 1176-154+154\sqrt{5}$ $= (1022+154\sqrt{5})\text{ cm}^2$	[1/2] [1] [1/2] [1] [1/2] [1] [1] [1/2]
22.	$Mode = \ell + \left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$ $= 60 + \left(\frac{29-21}{58-21-17}\right)\times 20$ = 68 So, the mode marks is 68. Empirical relationship between the three measures of central tendencies is: 3 Median = Mode + 2 Mean 3 Median = 68 + 2×53 Median = 58 marks	[1] [1] [1]

Section D

23.	Let original speed of the train be x km/h. Time taken at original speed = $\frac{360}{x}$ hours Time taken at increased speed = $\frac{360}{x+5}$ hours Now, $\frac{360}{x}-\frac{360}{x+5}=\frac{48}{60}$ $\Rightarrow 360\left[\frac{1}{x}-\frac{1}{x+5}\right]=\frac{4}{5}$ $\Rightarrow x^2 + 5x - 2250 = 0$ $\Rightarrow x = 45$ or -50 (as speed cannot be negative) $\Rightarrow x = 45$ km/h **OR** Discriminant = $b^2 - 4ac = 36 - 4 \times 5 \times (-2) = 76 > 0$ So, the given equation has two distinct real roots $5x^2 - 6x - 2 = 0$ Multiplying both sides by 5. $(5x)^2 - 2 \times (5x) \times 3 = 10$ $\Rightarrow (5x)^2 - 2 \times (5x) \times 3 + 3^2 = 10 + 3^2$ $\Rightarrow (5x - 3)^2 = 19$ $\Rightarrow 5x - 3 = \pm\sqrt{19}$ $\Rightarrow x = \frac{3\pm\sqrt{19}}{5}$ Verification: $5\left(\frac{3+\sqrt{19}}{5}\right)^2-6\left(\frac{3+\sqrt{19}}{5}\right)-2=\frac{9+6\sqrt{19}+19}{5}-\frac{18+6\sqrt{19}}{5}-\frac{10}{5}=0$ Similarly, $5\left(\frac{3-\sqrt{19}}{5}\right)^2-6\left(\frac{3-\sqrt{19}}{5}\right)-2=0$	[1] [1/2] [1½] [1] [1] [1] [1] [1/2] [1/2]
24.	Let the three middle most terms of the AP be a – d, a, a + d. We have, (a – d) + a + (a + d) = 225 $\Rightarrow 3a = 225 \Rightarrow a = 75$ Now, the AP is a – 18d,…,a – 2d, a – d, a, a + d, a + 2d,…, a + 18d Sum of last three terms: (a + 18d) + (a + 17d) + (a + 16d) = 429 $\Rightarrow 3a + 51d = 429 \Rightarrow a + 17d = 143$ $\Rightarrow 75 + 17d = 143$ $\Rightarrow d = 4$ Now, first term = a – 18d = 75 – 18(4) = 3 ∴ The AP is 3, 7, 11, …, 147.	[1] [1/2] [1] [1/2] [1]

25. Given: A right triangle ABC right angled at B.

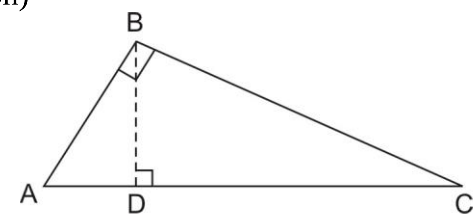
To prove: $AC^2 = AB^2 + BC^2$

Construction: Draw $BD \perp AC$ [1/2]

Proof: In Δ ADB and Δ ABC

$\angle ADB = \angle ABC$ (each 90°)

$\angle BAD = \angle CAB$ (common)



$\Delta ADB \sim \Delta ABC$ (By AA similarity criterion) [1]

Now, $\dfrac{AD}{AB} = \dfrac{AB}{AC}$ (corresponding sides are proportional)

$\Rightarrow AB^2 = AD \times AC$ …(i) [1]

Similarly $\Delta BDC \sim \Delta ABC$

$\Rightarrow BC^2 = CD \times AC$ …(ii) [1/2]

Adding (1) and (2)

$AB^2 + BC^2 = AD \times AC + CD \times AC$

$\Rightarrow AB^2 + BC^2 = AC \times (AD + CD)$

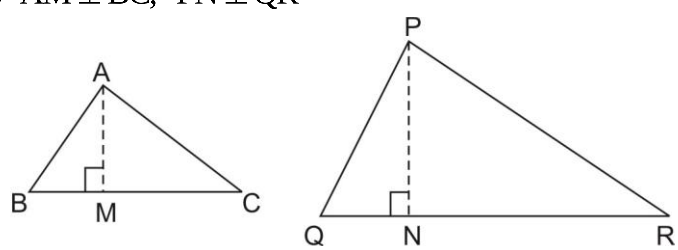
$\Rightarrow AB^2 + BC^2 = AC^2$, Hence Proved. [1]

OR

Given: $\Delta ABC \sim \Delta PQR$

To prove: $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\dfrac{AB}{PQ}\right)^2 = \left(\dfrac{BC}{QR}\right)^2 = \left(\dfrac{CA}{RP}\right)^2$

Construction: Draw $AM \perp BC$, $PN \perp QR$ [1/2]



$\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \dfrac{BC}{QR} \times \dfrac{AM}{PN}$ …(i) [1]

In Δ ABM and Δ PQN

$\angle B = \angle Q$ ($\because \Delta ABC \sim \Delta PQR$)

$\angle M = \angle N$ (each 90^o)

$\Delta ABM \sim \Delta PQN$ (AA similarity criterion)

Therefore, $\dfrac{AM}{PN} = \dfrac{AB}{PQ}$ …(ii) [1]

But $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{CA}{RP}$ ($\Delta ABC \sim \Delta PQR$) …(iii) [1/2]

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Hence, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC}{QR} \times \frac{AM}{PN}$ from (i)

$= \frac{AB}{PQ} \times \frac{AB}{PQ}$ [from (ii) and (iii)]

$= \left(\frac{AB}{PQ}\right)^2$

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$ Using (iii)

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[1/2]

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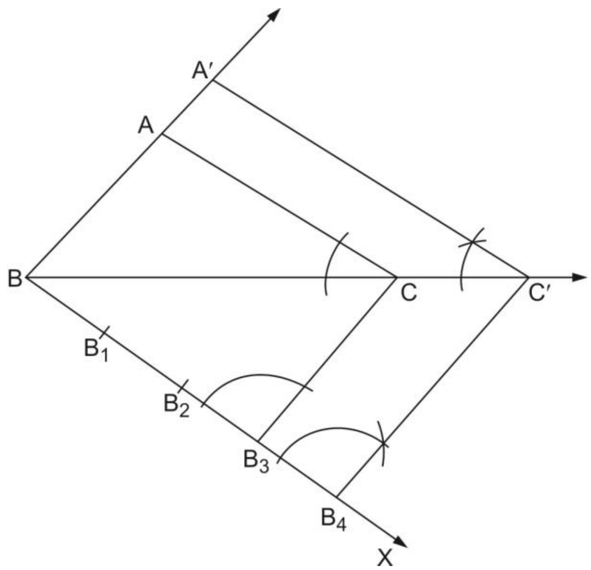
Draw Δ ABC in which BC = 7 cm, $\angle$B = 45°, $\angle$A = 105° and hence $\angle$C = 30°.
Construction of similar triangle A′BC′ as shown below:



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[3]

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<tr><td>27.</td><td>

LHS = $\frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1}$

$= \frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} \times \frac{\cos\theta + \sin\theta + 1}{\cos\theta + \sin\theta + 1}$

$= \frac{(\cos\theta + 1)^2 - \sin^2\theta}{(\cos\theta + \sin\theta)^2 - 1^2}$

$= \frac{\cos^2\theta + 1 + 2\cos\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1}$

$= \frac{2\cos^2\theta + 2\cos\theta}{2\sin\theta\cos\theta}$

$= \frac{2\cos\theta(\cos\theta + 1)}{2\sin\theta\cos\theta}$

$= \frac{\cos\theta + 1}{\sin\theta}$ = cosec θ + cot θ = RHS

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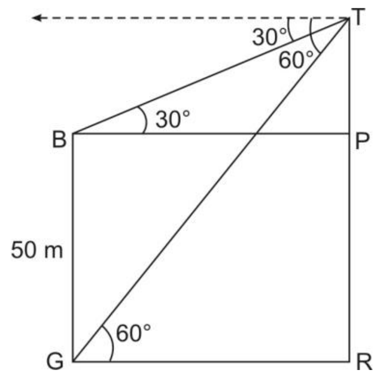
28.	In $\Delta BTP \Rightarrow \tan 30° = \frac{TP}{BP}$  Correct Figure $\Rightarrow \frac{1}{\sqrt{3}} = \frac{TP}{BP}$ $BP = TP\sqrt{3}$ …(i) In Δ GTR, $\tan 60° = \frac{TR}{GR} \Rightarrow \sqrt{3} = \frac{TR}{GR} \Rightarrow GR = \frac{TR}{\sqrt{3}}$ …(ii) Now, $TP\sqrt{3} = \frac{TR}{\sqrt{3}}$ (as BP = GR) $\Rightarrow$ 3TP = TP + PR $\Rightarrow$ 2TP = BG $\Rightarrow$ TP = $\frac{50}{2}$ m = 25 m Now, TR = TP + PR = (25 + 50) m. Height of tower =TR = 75 m. Distance between building and tower = GR = $\frac{TR}{\sqrt{3}}$ $\Rightarrow GR = \frac{75}{\sqrt{3}}$ m $= 25\sqrt{3}$ m	[1] [1/2] [1/2] [1] [1/2] [1/2]
29.	Capacity of mug (actual quantity of milk) = $\pi r^2 h - \frac{2}{3}\pi r^3$ $= \pi r^2\left(h - \frac{2}{3}r\right)$ $= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \left(14 - \frac{2}{3} \times \frac{7}{2}\right)$ $= \frac{2695}{6}$ cm^3 Amount dairy owner B should charge for one mug of milk $= \frac{2695}{6} \times \frac{80}{1000}$ = ₹ 35.93 Value exhibited by dairy owner B: **honesty** (or any similar value)	[1] [1] [1] [1]

30.

Daily pocket allowance (in ₹)	Number of children (f_i)	Mid-point (x_i)	$u_i = \frac{x_i - 18}{2}$	$f_i u_i$
11–13	3	12	–3	–9
13–15	6	14	–2	–12
15–17	9	16	–1	–9
17–19	13	18	0	0
19–21	k	20	1	k
21–23	5	22	2	10
23–25	4	24	3	12

$\Sigma f_i = 40 + k$ $\qquad$ $\Sigma f_i u_i = k - 8$ [2]

Mean $= \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$ [1]

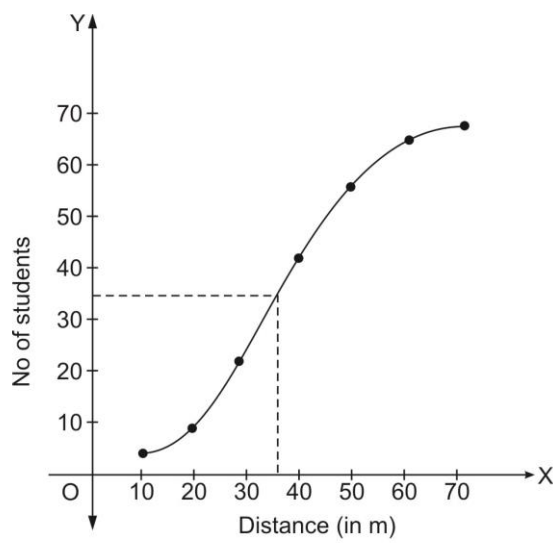
$18 = 18 + 2\left(\frac{k-8}{40+k}\right)$ [1]

$\Rightarrow \quad k = 8$

OR

Less than	Number of Students
10	4
20	9
30	22
40	42
50	56
60	64
70	68

[1]



Less than Ogive [2]

Median distance is value of x that corresponds to

Cumulative frequency $\frac{N}{2} = \frac{68}{2} = 34$

Therefore, Median distance = 36 m [1]