SAMPLE QUESTION PAPER

SQP 4

BLUEPRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	MCQs, A & R (1 mark)	VSA (2 marks)	SA (3 marks)	LA (5 marks)	Case Based (4 marks)	Total
1.	Relations and Functions	–	–	1(3)	–	–	**1(3)**
2.	Inverse Trigonometric Functions	1(1)	–	–	–	1(4)	**2(5)**
3.	Matrices	2(2)	–	–	–	–	**2(2)**
4.	Determinants	1(1)	1(2)	–	1(5)*	–	**3(8)**
5.	Continuity and Differentiability	2(2)	–	1(3)*	1(5)	1(4)*	**5(14)**
6.	Application of Derivatives	2(2)	1(2)*	–	–	–	**3(4)**
7.	Integrals	3(3)	1(2)	1(3)*	–	–	**5(8)**
8.	Application of Integrals	2(2)	–	–	–	–	**2(2)**
9.	Differential Equations	2(2)	1(2)	1(3)	–	–	**4(7)**
10.	Vector Algebra	1(1)	–	1(3)*	1(5)*	1(4)*	**4(13)**
11.	Three Dimensional Geometry	1(1)	–	–	–	–	**1(1)**
12.	Linear Programming	2(2)	–	1(3)	–	–	**3(5)**
13.	Probability	1(1)	1(2)*	–	1(5)	–	**3(8)**
	Total	**20(20)**	**5(10)**	**6(18)**	**4(20)**	**3(12)**	**38(80)**

*It is a choice based question.

Subject Code : **041**

SQP-4

MATHEMATICS

Time Allowed : 3 Hours ***Maximum Marks : 80***

General Instructions :

1. *This question paper contains - five sections* ***A, B, C, D*** *and* ***E****. Each section is compulsory. However, there are internal choices in some questions.*
2. ***Section A*** *has 18* ***MCQs and 02*** *Assertion-Reason based questions of 1 mark each.*
3. ***Section B*** *has 5* ***Very Short Answer (VSA)-type*** *questions of 2 marks each.*
4. ***Section C*** *has 6* ***Short Answer (SA)-type*** *questions of 3 marks each.*
5. ***Section D*** *has 4* ***Long Answer (LA)-type*** *questions of 5 marks each.*
6. ***Section E*** *has 3* ***source based/case based/passage based/integrated units of assessment*** *of 4 marks each with sub-parts.*

SECTION A

(Multiple Choice Questions)

Each question carries 1 mark

1. If $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$, then X = ?

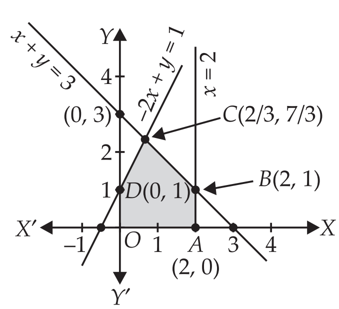
(a) $\begin{bmatrix}5 & 0\\ 0 & 4\end{bmatrix}$ (b) $\begin{bmatrix}7 & 0\\ 1 & 5\end{bmatrix}$ (c) $\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$ (d) $\begin{bmatrix}7 & 1\\ 0 & 4\end{bmatrix}$

2. Evaluate : $\int_{\pi/4}^{\pi/2} \cos 2x\, dx$

(a) 3/2 (b) 1/2

(c) −1/2 (d) −3/2

3. Based on the given shaded region as the feasible region in the graph, at how many point(s) is the objective function $Z = x + y$ minimum?



(a) a unique point (b) no point

(c) infinity many points (d) two points only

4. The area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$, $x = 3$ is

(a) $\frac{7}{2}$ sq. units (b) $\frac{9}{2}$ sq. units (c) $\frac{11}{2}$ sq. units (d) $\frac{13}{2}$ sq. units

5. If $\frac{3x+1}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$, then $\sin^{-1}\frac{A}{B} =$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

6. Determine the order and degree of $5\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}$.

(a) 3, 2 (b) 2, 2 (c) 1, 2 (d) 2, 1

7. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

(a) 2/7 (b) 3/7 (c) 1/7 (d) 5/7

8. The function $f(x) = (9 - x^2)^2$ increases in

(a) $(-3, 0) \cup (3, \infty)$ (b) $(-\infty, -3) \cup (3, \infty)$

(c) $(-\infty, -3) \cup (0, 3)$ (d) $(-\infty, 3)$

9. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then the value of x is

(a) 3 (b) 5 (c) 7 (d) 9

10. Find : $\int \frac{dx}{x^2 + 4x + 8}$

(a) $\frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right) + C$ (b) $\frac{1}{2}\tan^{-1}\left(\frac{x+3}{3}\right) + C$

(c) $\frac{1}{3}\tan^{-1}\frac{x+2}{3} + C$ (d) $\frac{1}{2}\tan^{-1}\left(\frac{x-3}{3}\right) + C$

11. Find the derivative of $y = \cos x^2$.

(a) $x \sin x^2$ (b) $\cos x \cdot 2x$

(c) $-2x \sin x^2$ (d) $2 \cos x^2$

12. The value of $f(0)$ so that $f(x) = \frac{(-e^x + 2^x)}{x}$ may be continuous at $x = 0$ is

(a) $\log\left(\frac{1}{2}\right)$ (b) 0

(c) 4 (d) $-1 + \log 2$

13. Find the values of a, b, c and d from the following equation :

$$\begin{bmatrix} 2a + b & a - 2b \\ 5c - d & 4c + 3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

(a) $a = 2, b = 1, c = 4, d = 3$ (b) $a = 1, b = 2, c = 3, d = 4$

(c) $a = 3, b = 4, c = 1, d = 2$ (d) $a = 4, b = 3, c = 2, d = 1$

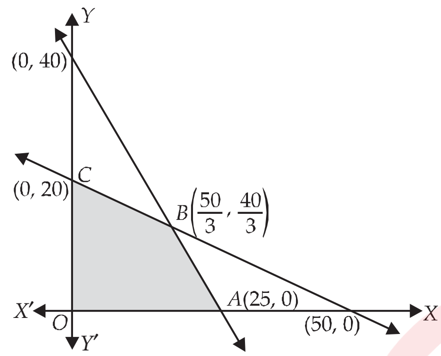
14. Find the integrating factor of the differential equation $\frac{dy}{dx}+(\sec x)y=\tan x$.

(a) $\sec x - \tan x$ (b) $\sec x + \tan x$

(c) $\sin x - \cos x$ (d) $\sin x + \cos x$

15. The feasible region for a LPP is shown in the figure.

Let $Z = x + y$ be the objective function, then the maximum value of Z occurs at



(a) a unique point (b) no point

(c) infinitely many points (d) two points only

16. Find the area bounded by the line $y = x$, x-axis and lines $x = -1$ to $x = 2$.

(a) 3/2 sq. units (b) 5/2 sq. units

(c) 1/2 sq. units (d) 7/2 sq. units

17. Evaluate : $\int_{-\pi/4}^{\pi/4} |\sin x|\,dx$

(a) $2-\sqrt{2}$ (b) $-\sqrt{2}+1$

(c) $-1+\sqrt{2}$ (d) $2+\sqrt{2}$

18. Write a unit vector in the direction of the vector $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$.

(a) $\frac{2}{3}\hat{i}-\frac{1}{3}\hat{j}+\frac{2}{3}\hat{k}$ (b) $-\frac{2}{3}\hat{i}+\frac{1}{3}\hat{j}+\frac{2}{3}\hat{k}$

(c) $\frac{1}{3}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$ (d) $\frac{2}{3}\hat{i}+\frac{1}{3}\hat{j}+\frac{2}{3}\hat{k}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

19. If $f'(x) = (x-1)^3(x-2)^8$, then

Assertion (A) : $f(x)$ has neither maximum nor minimum at $x = 2$.

Reason (R) : $f'(x)$ changes sign from negative to positive at $x = 2$.

20. **Assertion (A) :** The points (1, 2, 3), (–2, 3, 4) and (7, 9, 1) are collinear.

Reason (R) : If a line makes angle $\frac{\pi}{2}, \frac{3\pi}{4}, \frac{\pi}{4}$ with x, y and z respectively then its direction cosines are $0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$.

SECTION B

This section comprises of very short answer type questions (VSA) of 2 marks each

21. Show, that the function $f(x) = x^9 + 4x^7 + 11$ is increasing on R.

OR

Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

22. A machine produces parts that are either good (90%), slightly defective (2%), or obviously defective (8%). Produced parts get passed through an automatic inspection machine, which is able to detect any part that is obviously defective and discard it. What is the probability of the parts that make it through the inspection machine and get shipped?

OR

If A and B are two independent events such that $P\left(\bar{A} \cap B\right) = \frac{2}{15}$ and $P\left(A \cap \bar{B}\right) = \frac{1}{6}$, then find $P(B) - P(A)$.

23. Find the solution of the differential equation $\frac{dy}{dx} = \cos(x + y)$.

24. Write minors and cofactors of the elements of determinant $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$.

25. Evaluate : $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)}$

SECTION C

This section comprises of short answer type questions (SA) of 3 marks each

26. Check whether the relation R on the set $A = \{1, 2, 3\}$ defined as $R = \{(1, 1), (1, 2), (2, 1), (3, 3)\}$ is reflexive, symmetric and transitive.

27. Solve the following LPP graphically.

Minimize $Z = 5x + 7y$

Subject to constraints :

$2x + y \leq 8$

$x + 2y \geq 10$

and $x, y \geq 0$

28. Evaluate : $\int_0^{2\pi} \frac{dx}{e^{\sin x} + 1}$

OR

Using properties of definite integrals, evaluate the following : $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\tan x}} dx$

29. If $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{b}$ in the form $\vec{b} = \vec{b}_1 + \vec{b}_2$ where $\vec{b}_1 \,||\, \vec{a}$ and $\vec{b}_2 \perp \vec{a}$.

OR

If the shortest distance between the lines $L_1 : \frac{x-1}{1} = \frac{y}{-1} = \frac{z}{2}$ and $L_2 : \frac{x+1}{2} = \frac{y}{2} = \frac{z-3}{\lambda}$ is unity, then find the value of λ.

30. If $(ax + b)\, e^{y/x} = x$, then show that $x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

OR

Find $\frac{dy}{dx}$, when $x = a\left\{\cos t + \frac{1}{2}\log \tan^2 \frac{t}{2}\right\}$ and $y = a \sin t$.

31. Find the particular solution of differential equation $\cos y\, dx + (1 + e^{-x})\sin y\, dy = 0$, given that $y = \frac{\pi}{4}$, when $x = 0$.

SECTION D

This section comprises of long answer type questions (LA) of 5 marks each

32. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement, then find the probability that both drawn balls are black.

33. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then show that $\cos\frac{\theta}{2} = \frac{1}{2}|\vec{a}+\vec{b}|$.

OR

If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are distinct non-zero vectors represented by directed line segments from the origin to the points A, B, C and D respectively, and if $\vec{b} - \vec{a} = \vec{c} - \vec{d}$, then prove that $ABCD$ is a parallelogram.

34. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, then find AB. Hence, solve the system of equations :

$x - y = 6$, $2x + 3y + 4z = 34$, $y + 2z = 14$

OR

Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$ and verify that $A \cdot (\text{adj } A) = |A|\, I_3 = (\text{adj } A) \cdot A$.

35. If $f(x) = [x]$, $-2 \le x \le 2$, then show that $f(x)$ is neither continuous nor differentiable at $x = 1$.

SECTION E

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub-parts. The first two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. Read the following passage and answer the questions given below.

If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

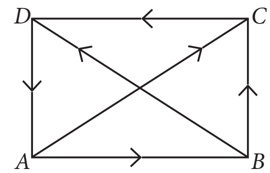
(i) If $\vec{p}, \vec{q}, \vec{r}$ are the vectors represented by the sides of a triangle taken in order, then find $\vec{q} + \vec{r}$.

(ii) If $ABCD$ is a parallelogram and AC and BD are its diagonals, then find $\overrightarrow{AC} + \overrightarrow{BD}$.

(iii) If $ABCD$ is a parallelogram, where $\overrightarrow{AB} = 2\vec{a}$ and $\overrightarrow{BC} = 2\vec{b}$, then find $\overrightarrow{AC} - \overrightarrow{BD}$.

OR

(iii) If $ABCD$ is a quadrilateral whose diagonals are $\overrightarrow{AC}$ and $\overrightarrow{BD}$, then find $\overrightarrow{BA} + \overrightarrow{CD}$.



37. Read the following passage and answer the questions given below.

If a relation between x and y is such that y cannot be expressed in terms of x, then y is called an implicit function of x. When a given relation expresses y as an implicit function of x and we want to find $\frac{dy}{dx}$, then we differentiate every term of the given relation w.r.t. x, remembering that a term in y is first differentiated w.r.t. y and then multiplied by $\frac{dy}{dx}$.

(i) Differentiate $x^3 + x^2y + xy^2 + y^3 = 81$ w.r.t. x.

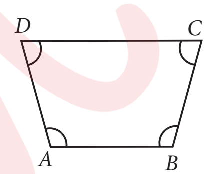
(ii) Differentiate $e^{\sin y} = xy$ w.r.t. x.

(iii) If $\sin^2 x + \cos^2 y = 1$, then find $\frac{dy}{dx}$.

OR

(iii) If $y = x \tan y$, then find $\frac{dy}{dx}$.

38. There are 4 friends A, B, C and D whose houses are situated at different places in a same colony and lines joining their houses forms a quadrilateral.



A mathematician measured angle between lines joining their houses as,

$$\angle A = \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$$

$$\angle B = \tan^{-1}(1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$$

$$\angle C = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\angle D = \cos^{-1}\left(\frac{1}{2}\right)$$

(i) Find the measure of angle A and angle B in terms of π.

(ii) Write the measure of angle C in degrees.

Self Evaluation Sheet

*Once you complete **SQP-4**, check your answers with the given solutions and fill your marks in the marks obtained column according to the marking scheme. Performance Analysis Table given at the bottom will help you to check your readiness.*

Q.No.	Chapter	Marks Per Question	Marks Obtained
1	Matrices	1	
2	Integrals	1	
3	Linear Programming	1	
4	Application of Integrals	1	
5	Inverse Trigonometric Functions	1	
6	Differential Equations	1	
7	Probability	1	
8	Application of Derivatives	1	
9	Determinants	1	
10	Integrals	1	
11	Continuity and Differentiability	1	
12	Continuity and Differentiability	1	
13	Matrices	1	
14	Differential Equations	1	
15	Linear Programming	1	
16	Application of Integrals	1	
17	Integrals	1	
18	Vector Algebra	1	
19	Application of Derivatives	1	
20	Three Dimensional Geometry	1	
21	Application of Derivatives / Application of Derivatives	2	
22	Probability / Probability	2	
23	Differential Equations	2	
24	Determinants	2	
25	Integrals	2	
26	Relations and Functions	3	
27	Linear Programming	3	
28	Integrals / Integrals	3	
29	Vector Algebra / Three Dimensional Geometry	3	
30	Continuity and Differentiability / Continuity and Differentiability	3	
31	Differential Equations	3	
32	Probability	5	
33	Vector Algebra / Vector Algebra	5	
34	Determinants / Determinants	5	
35	Continuity and Differentiability	5	
36	Vector Algebra	1 + 1 + 2	
37	Continuity and Differentiability	1 + 1 + 2	
38	Inverse Trigonometric Functions	2 + 2	
	Total	**80**	....
		Percentage	..............%

Performance Analysis Table

If your marks is

> 90%	**TREMENDOUS!**	➢ *You are done! Keep on revising to maintain the position.*
81-90%	**EXCELLENT!**	➢ *You have to take only one more step to reach the top of the ladder. Practise more.*
71-80%	**VERY GOOD!**	➢ *A little bit of more effort is required to reach the 'Excellent' bench mark.*
61-70%	**GOOD!**	➢ *Revise thoroughly and strengthen your concepts.*
51-60%	**FAIR PERFORMANCE!**	➢ *Need to work hard to get through this stage.*
40-50%	**AVERAGE!**	➢ *Try hard to boost your average score.*

SQP 4 SOLUTIONS

1. **(c) :** $(X+Y)+(X-Y)=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

$\Rightarrow\ 2X=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}\ \Rightarrow\ X=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$

2. **(c) :** We have, $I=\int_{\pi/4}^{\pi/2}\cos 2x\,dx=\left[\frac{\sin 2x}{2}\right]_{\pi/4}^{\pi/2}$

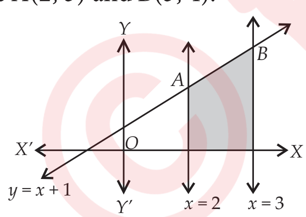
$=\frac{\sin\pi}{2}-\frac{\sin\frac{\pi}{2}}{2}=0-\frac{1}{2}=-\frac{1}{2}$

3. **(a) :**

Corner Points	Value of $Z = x + y$
$O(0, 0)$	0 (Minimum)
$A(2, 0)$	2
$B(2, 1)$	3
$C\left(\frac{2}{3},\frac{7}{3}\right)$	3
$D(0, 1)$	1

Since, minimum value of Z occurs at O. So, Z is minimum at a unique point.

4. **(a) :** We have $y = x + 1$ and lines $x = 2$, $x = 3$. Points of intersection are $A(2, 3)$ and $B(3, 4)$.



$\therefore$ Required area of the shaded region $=\int_2^3 (x+1)dx$

$=\left[\frac{x^2}{2}+x\right]_2^3=\left[\frac{9}{2}+3-\frac{4}{2}-2\right]=\frac{7}{2}$ sq. units

5. **(d) :** We have, $3x + 1 = A(x + 3) + B(x - 1)$

$\Rightarrow\ A + B = 3$ and $3A - B = 1$

On solving, we get $A = 1, B = 2$

$\therefore\ \sin^{-1}\frac{A}{B}=\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

6. **(b) :** The given differential equation can be written as

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3=\left(5\frac{d^2y}{dx^2}\right)^2$$

Clearly, it can be observed that order of differential equation is 2 and degree is 2.

7. **(b) :** Total number of students = 8

The number of ways to select 4 students out of 8 students

$={}^8C_4=\frac{8!}{4!\,4!}=70$

The number of ways to select 2 boys and 2 girls

$={}^3C_2\times{}^5C_2=\frac{3!}{2!1!}\times\frac{5!}{2!3!}=3\times10=30$

$\therefore$ Required probability $=\frac{30}{70}=\frac{3}{7}.$

8. **(a) :** Given, $f(x) = (9 - x^2)^2$

$\Rightarrow\ f'(x) = 2(9 - x^2)(-2x) = -4x(9 - x^2)$

For increasing $f'(x) \geq 0$

$\Rightarrow\ -4x(9 - x^2) \geq 0$

$-ve$ $+ve$ $-ve$ $+ve$

$-\infty$ -3 0 3 ∞

$\therefore\ f(x)$ increases in $(-3, 0) \cup (3, \infty)$.

9. **(d) :** We have, $\begin{vmatrix}5 & 3 & -1\\ -7 & x & -3\\ 9 & 6 & -2\end{vmatrix}=0$

$\Rightarrow\ 5(-2x + 18) - 3(14 + 27) - 1(-42 - 9x) = 0$

$\Rightarrow\ -x + 9 = 0 \Rightarrow x = 9$

10. **(a) :** We have, $\int\frac{dx}{x^2+4x+8}=\int\frac{dx}{x^2+4x+4+4}$

$=\int\frac{dx}{(x+2)^2+(2)^2}=\frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right)+C$

11. **(c) :** We have, $y = \cos x^2$

$\therefore\ \frac{dy}{dx}=-\sin x^2\cdot 2x=-2x\sin x^2$

12. **(d) :** $f(x)=\frac{-e^x+2^x}{x}$ and let $f(x)$ is continuous at $x = 0$.

Then, $\lim\limits_{x\to0}f(x)=f(0)\Rightarrow\lim\limits_{x\to0}\frac{-e^x+2^x}{x}=f(0)$

$\Rightarrow\ \lim\limits_{x\to0}\left\{-\left(\frac{e^x-1}{x}\right)+\left(\frac{2^x-1}{x}\right)\right\}=f(0)$

$\Rightarrow\ -1+\log 2=f(0)\Rightarrow f(0)=-1+\log 2$

13. (b) : By equality of two matrices, equating the corresponding elements, we get

$2a + b = 4$, $5c - d = 11$

$a - 2b = -3$, $4c + 3d = 24$

Solving these equations, we get

$a = 1$, $b = 2$, $c = 3$ and $d = 4$.

14. (b) : Given, $\frac{dy}{dx} + (\sec x)y = \tan x$,

which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$

Here, $P = \sec x$ and $Q = \tan x$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \sec x \, dx} = e^{\log|\sec x + \tan x|}$

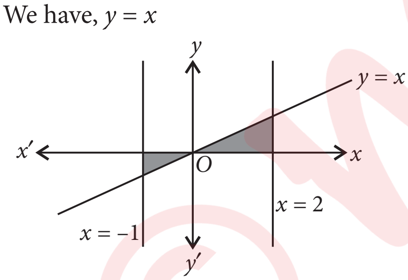
$= \sec x + \tan x$

15. (a) :

Corner Points	Value of $Z = x + y$
$O(0, 0)$	0
$A(25, 0)$	25
$B\left(\frac{50}{3}, \frac{40}{3}\right)$	30 (Maximum)
$C(0, 20)$	20

$\therefore$ Maximum value of Z occurs at a unique point.

16. (b) : We have, $y = x$



$\therefore$ Required area = area of shaded region

$$= \left|\int_{-1}^{0} x\,dx\right| + \left|\int_{0}^{2} x\,dx\right| = \left|\frac{x^2}{2}\right|_{-1}^{0} + \left|\frac{x^2}{2}\right|_{0}^{2}$$

$$= \left|-\frac{1}{2}\right| + |2| = 2 + \frac{1}{2} = \frac{5}{2} \text{ sq. units}$$

17. (a) : Let $I = \int_{-\pi/4}^{\pi/4} |\sin x|\,dx = 2\int_{0}^{\pi/4} \sin x\,dx$

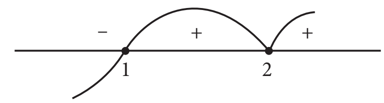
$$= 2[-\cos x]_0^{\pi/4} = -2\left[\frac{1}{\sqrt{2}} - 1\right] = 2 - \sqrt{2}$$

18. (d) : We have, $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$

$\therefore$ $|\vec{a}| = \sqrt{(2)^2 + (1)^2 + (2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$

Required unit vector is $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$

19. (c) : It is clear from figure that $f'(x)$ has no sign change at $x = 2$.



Hence, $f(x)$ has neither maximum nor minimum at $x = 2$.

20. (b) : Both A are R true but R is not the correct explanation of A.

21. Here, $f(x) = x^9 + 4x^7 + 11$

$\therefore$ $f'(x) = 9x^8 + 28x^6 = x^6(9x^2 + 28) > 0$ for all $x \in R$

Thus, $f(x)$ is increasing on R.

OR

Let V be the volume of a closed cuboid with length x, breadth x and height y. Let S be the surface area of cuboid. Then

$x^2y = V$ and $S = 2(x^2 + xy + xy) = 2(x^2 + 2xy)$

$\therefore$ $S = 2\left[x^2 + 2x \cdot \frac{V}{x^2}\right] = 2\left[x^2 + \frac{2V}{x}\right]$

$\therefore$ $\frac{dS}{dx} = 2\left[2x - \frac{2V}{x^2}\right] = 0$

$\Rightarrow$ $x^3 = V = x^2y \Rightarrow x = y$

Now, $\frac{d^2S}{dx^2} = 2\left[2 + \frac{4V}{x^3}\right] > 0$

$\therefore$ $x = y$ will give minimum surface area

and $x = y$ means all the sides are equal.

$\therefore$ Cube will have minimum surface area.

22. Let G, SD, OD be the events that a randomly chosen part is good, slightly defective, obviously defective respectively.

Then, $P(G) = 0.90$, $P(SD) = 0.02$, and $P(OD) = 0.08$

Required probability $= P(G \mid OD^c)$

$$= \frac{P(G \cap OD^c)}{P(OD^c)} = \frac{P(G)}{1 - P(OD)} = \frac{0.90}{1 - 0.08} = \frac{90}{92} = 0.978$$

OR

Since A and B are independent events, therefore, $\bar{A}$ and B are independent and also A and $\bar{B}$ are independent.

$\therefore$ $P(\bar{A} \cap B) = P(\bar{A})\,P(B) = (1 - P(A))\,P(B)$

and $P(A \cap \bar{B}) = P(A)\,P(\bar{B}) = P(A)\,(1 - P(B))$

$\Rightarrow$ $(1 - P(A))\,P(B) = P(\bar{A} \cap B) = \frac{2}{15}$ ...(i)

and $P(A)\,(1 - P(B)) = P(A \cap \bar{B}) = \frac{1}{6}$ ...(ii)

Subtracting (ii) from (i), we obtain

$$P(B) - P(A) = \frac{2}{15} - \frac{1}{6} = \frac{4-5}{30} = -\frac{1}{30}$$

23. We have $\frac{dy}{dx}=\cos(x+y)$

Put $x+y=u \Rightarrow \frac{dy}{dx}=\frac{du}{dx}-1$

$$\frac{du}{dx}-1=\cos u \Rightarrow \int\frac{du}{1+\cos u}=\int dx+c$$

$$\Rightarrow \int\frac{1}{2}\sec^2\left(\frac{u}{2}\right)du=x+c \Rightarrow \tan\left(\frac{u}{2}\right)=x+c$$

$$\Rightarrow \tan\left(\frac{x+y}{2}\right)=x+c$$

24. Let $A=\begin{vmatrix}1 & 0 & 4\\ 3 & 5 & -1\\ 0 & 1 & 2\end{vmatrix}$.

$$M_{11}=\begin{vmatrix}5 & -1\\ 1 & 2\end{vmatrix}=10+1=11,\ M_{12}=\begin{vmatrix}3 & -1\\ 0 & 2\end{vmatrix}=6,\ M_{13}=\begin{vmatrix}3 & 5\\ 0 & 1\end{vmatrix}=3$$

$$M_{21}=\begin{vmatrix}0 & 4\\ 1 & 2\end{vmatrix}=-4,\ M_{22}=\begin{vmatrix}1 & 4\\ 0 & 2\end{vmatrix}=2,\ M_{23}=\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix}=1$$

$$M_{31}=\begin{vmatrix}0 & 4\\ 5 & -1\end{vmatrix}=-20,\ M_{32}=\begin{vmatrix}1 & 4\\ 3 & -1\end{vmatrix}=-13,\ M_{33}=\begin{vmatrix}1 & 0\\ 3 & 5\end{vmatrix}=5$$

For cofactors, we know that $C_{ij}=(-1)^{i+j}M_{ij}$,
$C_{11}=11$, $C_{12}=-6$, $C_{13}=3$, $C_{21}=4$, $C_{22}=2$,
$C_{23}=-1$, $C_{31}=-20$, $C_{32}=13$, $C_{33}=5$

25. Let $I=\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)}$

$$=\frac{1}{5}\left(\int_0^\infty\frac{1}{x^2+4}dx-\int_0^\infty\frac{1}{(x^2+9)}dx\right)$$

$$=\frac{1}{5}\left[\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^\infty-\left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_0^\infty\right]$$

$$=\frac{1}{5}\left[\left(\frac{1}{2}\cdot\frac{\pi}{2}-0\right)-\left(\frac{1}{3}\cdot\frac{\pi}{2}-0\right)\right]=\frac{1}{5}\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\frac{\pi}{60}$$

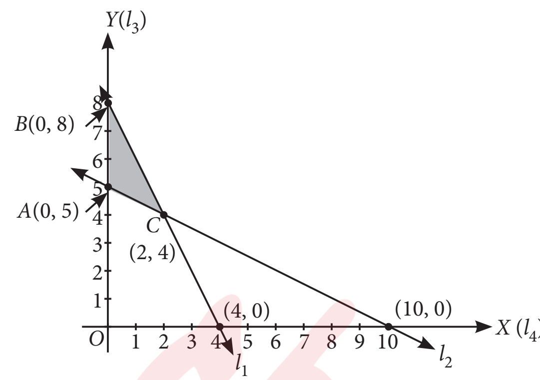
26. Given, $R=\{(1,1),(1,2),(2,1),(3,3)\}$
Since, $(2,2)\notin R$
Therefore, R is not reflexive.
But R is both symmetric and transitive as $(1,2)\in R$
$\Rightarrow$ $(2,1)\in R$ and $(1,1)\in R$, $(1,2)\in R$
$\Rightarrow$ $(1,1)\in R$.

27. The given problem is
Minimize $Z=5x+7y$
subject to $2x+y\le 8$
$x+2y\ge 10$ and $x, y\ge 0$
To solve this LPP graphically, we first convert the inequations into equations to obtain the following line

$l_1: 2x+y=8$, or $\frac{x}{4}+\frac{y}{8}=1$

$l_2: x+2y=10$, or $\frac{x}{10}+\frac{y}{5}=1$

$l_3: x=0$ and $l_4: y=0$



The coordinates of the corner points of the feasible region ABC are $A(0,5)$, $B(0,8)$ and $C(2,4)$.
The values of the objective function $Z=5x+7y$ at the corner points of the feasible region are given in the following table.

Corner Points	Value of $Z=5x+7y$
$A(0,5)$	$5\times0+7\times5=35$ (Minimum)
$B(0,8)$	$5\times0+7\times8=56$
$C(2,4)$	$5\times2+7\times4=38$

Thus, Z is minimum when $x=0$ and $y=5$.

28. Let $I=\int_0^{2\pi}\frac{dx}{e^{\sin x}+1}$ ...(i)

$$\Rightarrow I=\int_0^{2\pi}\frac{dx}{e^{\sin(2\pi-x)}+1}\qquad \left(\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{2\pi}\frac{dx}{e^{-\sin x}+1}\ \Rightarrow\ I=\int_0^{2\pi}\frac{e^{\sin x}}{e^{\sin x}+1}dx \qquad ...(ii)$$

Adding (i) and (ii), we get

$$2I=\int_0^{2\pi}1\cdot dx=2\pi\quad \therefore\quad I=\pi$$

OR

Let $I=\int_{\pi/6}^{\pi/3}\frac{dx}{(1+\sqrt{\tan x})}$

$$\Rightarrow I=\int_{\pi/6}^{\pi/3}\frac{dx}{\left(1+\sqrt{\frac{\sin x}{\cos x}}\right)}=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \qquad ...(1)$$

$$\Rightarrow \quad I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}dx$$

$$\left[\because \int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$$

$$\Rightarrow \quad I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$$

$$\Rightarrow \quad I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \qquad \text{...(2)}$$

Adding (1) and (2), we get

$$2I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx=\int_{\pi/6}^{\pi/3}dx$$

$$\Rightarrow \quad 2I=[x]_{\pi/6}^{\pi/3}=\left(\frac{\pi}{3}-\frac{\pi}{6}\right)=\frac{\pi}{6}$$

$$\Rightarrow \quad I=\frac{\pi}{12}$$

29. Here $\vec{a}=3\hat{i}-\hat{j},\ \vec{b}=2\hat{i}+\hat{j}-3\hat{k}$

We have to express : $\vec{b}=\vec{b}_1+\vec{b}_2$, where

$\vec{b}_1 \parallel \vec{a}$ and $\vec{b}_2 \perp \vec{a}$

Let $\vec{b}_1=\lambda\vec{a}=\lambda(3\hat{i}-\hat{j})$ and $\vec{b}_2=x\hat{i}+y\hat{j}+z\hat{k}$

Now $\vec{b}_2 \perp \vec{a} \Rightarrow \vec{b}_2\cdot\vec{a}=0$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k})\cdot(3\hat{i}-\hat{j})=0$

$\Rightarrow \quad 3x-y=0$ ...(i)

Now, $\vec{b}=\vec{b}_1+\vec{b}_2$

$\Rightarrow 2\hat{i}+\hat{j}-3\hat{k}=\lambda(3\hat{i}-\hat{j})+(x\hat{i}+y\hat{j}+z\hat{k})$

On comparing, we get

$$\left.\begin{array}{l}2=3\lambda+x\\ 1=-\lambda+y\end{array}\right]\Rightarrow x+3y=5 \qquad \text{...(ii)}$$

and $-3=z \Rightarrow z=-3$

Solving (i) and (ii), we get $x=\frac{1}{2}, y=\frac{3}{2}$

$\therefore 1=-\lambda+y \Rightarrow 1=-\lambda+\frac{3}{2}\Rightarrow \lambda=\frac{1}{2}$

Hence, $\vec{b}_1=\lambda(3\hat{i}-\hat{j})=\frac{3}{2}\hat{i}-\frac{1}{2}\hat{j}$

and $\vec{b}_2=\frac{1}{2}\hat{i}+\frac{3}{2}\hat{j}-3\hat{k}$

OR

The lines are $\frac{x-1}{1}=\frac{y}{-1}=\frac{z}{2}$ ...(i)

and $\frac{x+1}{2}=\frac{y}{2}=\frac{z-3}{\lambda}$ ...(ii)

Here, $x_1=1, y_1=0, z_1=0$

$a_1=1, b_1=-1, c_1=2$

$x_2=-1, y_2=0, z_2=3$

$a_2=2, b_2=2, c_2=\lambda$

Now, $\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}=\begin{vmatrix}-2 & 0 & 3\\ 1 & -1 & 2\\ 2 & 2 & \lambda\end{vmatrix}$

$=-2(-\lambda-4)+0+3(2+2)$

$=2\lambda+8+12=2\lambda+20$

and $(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2$

$=(-\lambda-4)^2+(4-\lambda)^2+(2+2)^2$

$=\lambda^2+16+8\lambda+16+\lambda^2-8\lambda+16=2\lambda^2+48$

Shortest distance between lines

$=\frac{|2\lambda+20|}{\sqrt{2\lambda^2+48}}=1$ [Given]

$\Rightarrow \ |2\lambda+20|=\sqrt{2\lambda^2+48}$

$\Rightarrow \ 4\lambda^2+400+80\lambda=2\lambda^2+48$

$\Rightarrow \ 2\lambda^2+80\lambda+352=0 \Rightarrow \lambda^2+40\lambda+176=0$

$$\Rightarrow \lambda=\frac{-40\pm\sqrt{1600-4(1)(176)}}{2}$$

$$=\frac{-40\pm\sqrt{1600-704}}{2}=\frac{-40\pm\sqrt{896}}{2}$$

$$=\frac{-40\pm2\sqrt{224}}{2}=-20\pm\sqrt{224}$$

30. Given, $(ax+b)e^{y/x}=x$

$\Rightarrow \ e^{y/x}=\frac{x}{ax+b}$

Taking log on both sides, we get

$\frac{y}{x}\cdot\log e=\log\frac{x}{ax+b}$

$\Rightarrow \ \frac{y}{x}=\log x-\log(ax+b)$ $(\because \log e=1)$

Differentiating w.r.t. x, we get

$$\frac{x\cdot\frac{dy}{dx}-y\cdot 1}{x^2}=\frac{1}{x}-\frac{1}{ax+b}\cdot a$$

$$\Rightarrow \ x\frac{dy}{dx}-y=x^2\cdot\frac{ax+b-ax}{x(ax+b)}$$

$\Rightarrow \quad x\dfrac{dy}{dx} - y = \dfrac{bx}{ax+b}$ ...(i)

Differentiating again w.r.t. x, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot 1 - \frac{dy}{dx} = \frac{(ax+b)\cdot b - bx\cdot a}{(ax+b)^2}$$

$$\Rightarrow \quad x\frac{d^2y}{dx^2} = \frac{b^2}{(ax+b)^2} \quad \Rightarrow x^3\frac{d^2y}{dx^2} = \left(\frac{bx}{ax+b}\right)^2$$

$$\Rightarrow \quad x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2 \qquad \text{(Using (i))}$$

OR

We have, $x = a\left\{\cos t + \dfrac{1}{2}\log\tan^2\dfrac{t}{2}\right\}$ and $y = a\sin t$

$$\Rightarrow \quad x = a\left\{\cos t + \frac{1}{2}\cdot 2\log\tan\frac{t}{2}\right\}$$

$$\Rightarrow \quad x = a\left\{\cos t + \log\tan\frac{t}{2}\right\}$$

Differentiating w.r.t. t, we get

$$\frac{dx}{dt} = a\left\{-\sin t + \frac{1}{\tan t/2}\sec^2\frac{t}{2}\cdot\frac{1}{2}\right\} \text{ and } \frac{dy}{dt} = a\cos t$$

$$\Rightarrow \quad \frac{dx}{dt} = a\left\{-\sin t + \frac{1}{2\sin(t/2)\cos(t/2)}\right\}$$

$$\Rightarrow \quad \frac{dx}{dt} = a\left\{-\sin t + \frac{1}{\sin t}\right\} \Rightarrow \frac{dx}{dt} = a\left\{\frac{-\sin^2 t + 1}{\sin t}\right\}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{a\cos^2 t}{\sin t}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a\cos t}{\dfrac{a\cos^2 t}{\sin t}} = \tan t$$

31. We have, $\cos y\, dx + (1 + e^{-x})\sin y\, dy = 0$

$\Rightarrow dx + (1 + e^{-x})\tan y\, dy = 0 \Rightarrow \dfrac{dx}{1+e^{-x}} + \tan y\, dy = 0$

Integrating on both sides, we get

$$\int\frac{e^x}{1+e^x}dx + \int\tan y\, dy = 0$$

$\Rightarrow \log(1 + e^x) + \log|\sec y| = \log C \Rightarrow \sec y\,(1 + e^x) = C$

When, $x = 0$, $y = \dfrac{\pi}{4}$, we get $C = 2\sqrt{2}$

$\therefore$ Particular solution of the differential equation is, $\sec y(1 + e^x) = 2\sqrt{2}$

32. Let E and F denote respectively the events that first and second ball drawn are black. We have to find $P(E \cap F)$ or $P(EF)$.

Now, $P(E) = P(\text{black ball in first draw}) = \dfrac{10}{15}$

When second ball is drawn without replacement, the probability that the second ball is black is the conditional probability of event F occurring when event E has already occurred.

$\therefore \quad P(F\,|\,E) = \dfrac{9}{14}$

By multiplication rule of probability, we have

$P(E \cap F) = P(E)\cdot P(F|E) = \dfrac{10}{15}\times\dfrac{9}{14} = \dfrac{3}{7}$

33. Since $\vec{a}$ and $\vec{b}$ are unit vectors

$\therefore \; |\vec{a}| = |\vec{b}| = 1$

We have,

$$|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = |\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2$$

$$\Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2|\vec{a}||\vec{b}|\cos\theta + |\vec{b}|^2$$

$$\left[\because \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta\right]$$

$$\Rightarrow |\vec{a}+\vec{b}|^2 = 2 + 2\cos\theta \qquad \left[\because |\vec{a}| = |\vec{b}| = 1\right]$$

$$\Rightarrow |\vec{a}+\vec{b}|^2 = 2(1+\cos\theta) = 2\left(2\cos^2\frac{\theta}{2}\right)$$

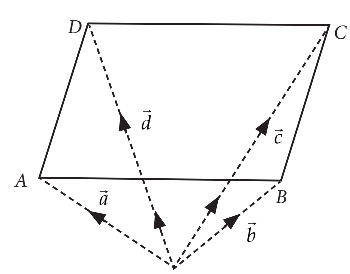
$$\left[\because 1+\cos\theta = 2\cos^2\frac{\theta}{2}\right]$$

$$\Rightarrow 4\cos^2\frac{\theta}{2} = |\vec{a}+\vec{b}|^2 \Rightarrow \cos^2\frac{\theta}{2} = \frac{1}{4}|\vec{a}+\vec{b}|^2$$

$$\Rightarrow \cos\frac{\theta}{2} = \frac{1}{2}|\vec{a}+\vec{b}|$$

OR

Let O be the origin. It is given that $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$ and $\overrightarrow{OD} = \vec{d}$, such that $\vec{b} - \vec{a} = \vec{c} - \vec{d}$.



$\Rightarrow \quad \overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{OC} - \overrightarrow{OD}$ ...(i)

In ΔOAB, we have

$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$\Rightarrow \quad \overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB}$ ...(ii)

In ΔOCD, we have

$\overrightarrow{OC} + \overrightarrow{CD} = \overrightarrow{OD}$

$\Rightarrow \quad \overrightarrow{OC} - \overrightarrow{OD} = -\overrightarrow{CD} \Rightarrow \overrightarrow{OC} - \overrightarrow{OD} = \overrightarrow{DC}$ ...(iii)

From (i), (ii) and (iii), we get $\overrightarrow{AB} = \overrightarrow{DC}$

$\therefore$ AB and CD are parallel and equal.

Now, $\overrightarrow{OC}-\overrightarrow{OB}=\overrightarrow{OD}-\overrightarrow{OA}$ (From (i)) ...(iv)

and $\overrightarrow{OC}-\overrightarrow{OB}=\overrightarrow{BC}$ and $\overrightarrow{OD}-\overrightarrow{OA}=\overrightarrow{AD}$ ...(v)

Using (iv) & (v), we get $\overrightarrow{BC}=\overrightarrow{AD}$

$\therefore$ BC and AD are parallel and equal.

Hence, $ABCD$ is a parallelogram.

34. $AB=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$

$=\begin{bmatrix}2+4+0 & 2-2+0 & -4+4+0\\ 4-12+8 & 4+6-4 & -8-12+20\\ 0-4+4 & 0+2-2 & 0-4+10\end{bmatrix}=\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$

$=6\,I_3$

$\Rightarrow\ A\left(\frac{1}{6}B\right)=I_3\Rightarrow A^{-1}=\frac{1}{6}B$

(By definition of inverse)

$\Rightarrow\ A^{-1}=\frac{1}{6}\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$

The given system of equations is

$x-y+0z=6$

$2x+3y+4z=34$

$0x+y+2z=14$

This system of equations can be written as

$\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}6\\ 34\\ 14\end{bmatrix}$ or $AX=C$

where $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $C=\begin{bmatrix}6\\ 34\\ 14\end{bmatrix}$

As A^{-1} exists, therefore $X=A^{-1}C$

$\Rightarrow\ X=\frac{1}{6}\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}\begin{bmatrix}6\\ 34\\ 14\end{bmatrix}$

$\Rightarrow\ \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{6}\begin{bmatrix}12+68-56\\ -24+68-56\\ 12-34+70\end{bmatrix}=\frac{1}{6}\begin{bmatrix}24\\ -12\\ 48\end{bmatrix}=\begin{bmatrix}4\\ -2\\ 8\end{bmatrix}$

$\Rightarrow\ x=4,\ y=-2,\ z=8$

Hence, the solution of the given system of equations is $x=4,\ y=-2,\ z=8$.

OR

We have the matrix $A=\begin{bmatrix}1 & 0 & -1\\ 3 & 4 & 5\\ 0 & -6 & -7\end{bmatrix}$

$\therefore\ C_{11}=\begin{vmatrix}4 & 5\\ -6 & -7\end{vmatrix}=2,\ C_{12}=-\begin{vmatrix}3 & 5\\ 0 & -7\end{vmatrix}=21,$

$C_{13}=\begin{vmatrix}3 & 4\\ 0 & -6\end{vmatrix}=-18,$

$C_{21}=-\begin{vmatrix}0 & -1\\ -6 & -7\end{vmatrix}=6,\ C_{22}=\begin{vmatrix}1 & -1\\ 0 & -7\end{vmatrix}=-7,$

$C_{23}=-\begin{vmatrix}1 & 0\\ 0 & -6\end{vmatrix}=6,$

$C_{31}=\begin{vmatrix}0 & -1\\ 4 & 5\end{vmatrix}=4,\ C_{32}=-\begin{vmatrix}1 & -1\\ 3 & 5\end{vmatrix}=-8,\ C_{33}=\begin{vmatrix}1 & 0\\ 3 & 4\end{vmatrix}=4$

$\therefore\ \text{adj } A=\begin{bmatrix}C_{11} & C_{21} & C_{31}\\ C_{12} & C_{22} & C_{32}\\ C_{13} & C_{23} & C_{33}\end{bmatrix}=\begin{bmatrix}2 & 6 & 4\\ 21 & -7 & -8\\ -18 & 6 & 4\end{bmatrix}$

and $|A|=\begin{vmatrix}1 & 0 & -1\\ 3 & 4 & 5\\ 0 & -6 & -7\end{vmatrix}=20$

Now, $A\cdot(\text{adj } A)=\begin{bmatrix}1 & 0 & -1\\ 3 & 4 & 5\\ 0 & -6 & -7\end{bmatrix}\begin{bmatrix}2 & 6 & 4\\ 21 & -7 & -8\\ -18 & 6 & 4\end{bmatrix}$

$=\begin{bmatrix}20 & 0 & 0\\ 0 & 20 & 0\\ 0 & 0 & 20\end{bmatrix}=20I_3=|A|I_3$

Similarly, we can obtain $(\text{adj } A)\cdot A=|A|I_3$

35. Given, $f(x)=\begin{cases}-2, \text{ if } -2\le x<-1\\ -1, \text{ if } -1\le x<0\\ 0, \text{ if } 0\le x<1\\ 1, \text{ if } 1\le x<2\\ 2, \text{ if } 2\le x\end{cases}$

Clearly, $f(1)=1$

$\lim\limits_{x\to 1^-} f(x)=\lim\limits_{h\to 0^+} f(1-h)=\lim\limits_{h\to 0^+}(0)=0$

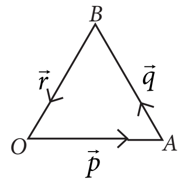
$\lim\limits_{x\to 1^+} f(x)=\lim\limits_{h\to 0^+} f(1+h)=\lim\limits_{h\to 0^+}(1)=1$

$\because\ \lim\limits_{x\to 1^-} f(x)\ne\lim\limits_{x\to 1^+} f(x)=f(1)$

$\therefore$ $f(x)$ is not continuous at $x=1$ and hence non differentiable at $x=1$.

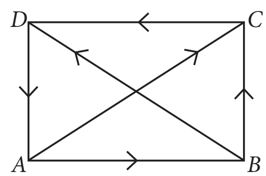
[$\because$ Every differentiable function is continuous]

36. (i) Let OAB be a triangle such that
$\overrightarrow{AO}=-\vec{p}, \overrightarrow{AB}=\vec{q}, \overrightarrow{BO}=\vec{r}$



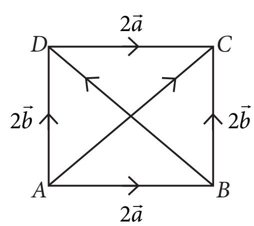
Now, $\vec{q}+\vec{r}=\overrightarrow{AB}+\overrightarrow{BO} = \overrightarrow{AO}=-\vec{p}$

(ii) From triangle law of vector addition,
$\overrightarrow{AC}+\overrightarrow{BD}=\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{BC}+\overrightarrow{CD}$



$=\overrightarrow{AB}+2\overrightarrow{BC}+\overrightarrow{CD}$
$=\overrightarrow{AB}+2\overrightarrow{BC}-\overrightarrow{AB}=2\overrightarrow{BC}$ $\quad [\because \overrightarrow{AB}=-\overrightarrow{CD}]$

(iii) In ΔABC, $\overrightarrow{AC}=2\vec{a}+2\vec{b}$ ...(i)



and in ΔABD, $2\vec{b}=2\vec{a}+\overrightarrow{BD}$ ...(ii)
[By triangle law of addition]

Adding (i) and (ii), we have
$$\overrightarrow{AC}+2\vec{b}=4\vec{a}+\overrightarrow{BD}+2\vec{b}$$
$$\Rightarrow \overrightarrow{AC}-\overrightarrow{BD}=4\vec{a}$$

OR

(iii) In ΔABC, $\overrightarrow{BA}+\overrightarrow{AC}=\overrightarrow{BC}$ ...(i)
[By triangle law]

In ΔBCD, $\overrightarrow{BC}+\overrightarrow{CD}=\overrightarrow{BD}$ ...(ii)

From (i) and (ii), $\overrightarrow{BA}+\overrightarrow{AC}=\overrightarrow{BD}-\overrightarrow{CD}$
$\Rightarrow \overrightarrow{BA}+\overrightarrow{CD}=\overrightarrow{BD}-\overrightarrow{AC}=\overrightarrow{BD}+\overrightarrow{CA}$

37. (i) $x^3+x^2y+xy^2+y^3=81$
$$\Rightarrow 3x^2+x^2\frac{dy}{dx}+2xy+2xy\frac{dy}{dx}+y^2+3y^2\frac{dy}{dx}=0$$
$$\Rightarrow (x^2+2xy+3y^2)\frac{dy}{dx}=-3x^2-2xy-y^2$$
$$\Rightarrow \frac{dy}{dx}=\frac{-(3x^2+2xy+y^2)}{x^2+2xy+3y^2}$$

(ii) $e^{\sin y}=xy \Rightarrow \sin y=\log x+\log y$
$$\Rightarrow \cos y\frac{dy}{dx}=\frac{1}{x}+\frac{1}{y}\frac{dy}{dx} \Rightarrow \frac{dy}{dx}\left[\cos y-\frac{1}{y}\right]=\frac{1}{x}$$
$$\Rightarrow \frac{dy}{dx}=\frac{y}{x(y\cos y-1)}$$

(iii) $\sin^2x+\cos^2y=1$
$$\Rightarrow 2\sin x\cos x+2\cos y\left(-\sin y\frac{dy}{dx}\right)=0$$
$$\Rightarrow \frac{dy}{dx}=\frac{-\sin 2x}{-\sin 2y}=\frac{\sin 2x}{\sin 2y}$$

OR

(iii) Here, $y=x\tan y$ ...(i)
Differentiating (i) both sides w.r.t. x, we get
$$\frac{dy}{dx}=1\cdot\tan y+x\cdot\sec^2 y\cdot\frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx}(1-x\sec^2 y)=\tan y$$
$$\Rightarrow \frac{dy}{dx}=\frac{\tan y}{1-x\sec^2 y}=\frac{\tan y}{1-x(1+\tan^2 y)} \quad \text{[From (i)]}$$
$$=\frac{y/x}{1-x\left(1+\frac{y^2}{x^2}\right)}=\frac{y}{x-x^2-y^2}$$

38. (i) We know, $\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}\in[0,\pi]$
$$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$$
$$\therefore \cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}+\frac{2\pi}{6}=\frac{2\pi}{3}$$
Also, $\tan^{-1}(1)=\frac{\pi}{4}\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

$\therefore$ The value of $\tan^{-1}(1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right)$
$$=\frac{\pi}{4}+\frac{\pi}{3}+\frac{\pi}{6}=\frac{3\pi}{4}$$
$$\therefore \angle A=\frac{2\pi}{3} \text{ and } \angle B=\frac{3\pi}{4}$$

(ii) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

So, the value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}=45^\circ$

Also, $\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}\in[0,\pi]$

So, the value of $\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}=60^\circ$

$\therefore \angle C=45^\circ$ and $\angle D=60^\circ$

☺☺☺