SOLUTIONS

1. $\int \frac{\cos(x+a)}{\sin(x+b)}dx = \int \frac{\cos(x+b+a-b)}{\sin(x+b)}dx$

$= \int \frac{\cos(x+b)\cos(a-b)-\sin(x+b)\sin(a-b)}{\sin(x+b)}dx$

$= \cos(a-b)\int \frac{\cos(x+b)}{\sin(x+b)}dx - \sin(a-b)\int dx$

$= \cos(a-b)\log\sin(x+b) - x\sin(a-b) + C$

2. Order = 2, Degree = 3

$\therefore$ Required sum = 2 + 3 = 5

OR

We have, $x\frac{dy}{dx}+2y = x^2 \Rightarrow \frac{dy}{dx}+2\frac{y}{x} = x$

$\therefore$ I.F. $= e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$

3. We have, $l=\cos\frac{\pi}{3}=\frac{1}{2}, m=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ and $n=\cos\theta$

Now, $l^2+m^2+n^2=1$

$\Rightarrow \left(\frac{1}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+n^2=1$

$\Rightarrow \frac{1}{4}+\frac{1}{2}+n^2=1 \Rightarrow n^2=\frac{1}{4} \Rightarrow n=\pm\frac{1}{2}$

$\Rightarrow \cos\theta=\pm\frac{1}{2}$

But θ is an acute angle (given).

$\therefore \theta=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$

4. We have, $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{2}$,

Also, $P(A \cap C) = \frac{1}{5}$ and $P(B \cap C) = \frac{1}{4}$

$\therefore P(C/B) = \frac{P(C\cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$

and, $P(\bar{A}\cap\bar{C}) = P(\overline{A\cup C})$

$= 1 - \{P(A) + P(C) - P(A \cap C)\} = 1 - \left(\frac{2}{5}+\frac{1}{2}-\frac{1}{5}\right) = \frac{3}{10}$

5. The given line is $5x - 3 = 15y + 7 = 3 - 10z$

$\Rightarrow \frac{x-\frac{3}{5}}{\frac{1}{5}} = \frac{y+\frac{7}{15}}{\frac{1}{15}} = \frac{z-\frac{3}{10}}{-\frac{1}{10}}$

Its direction ratios are $\frac{1}{5}, \frac{1}{15}, -\frac{1}{10}$

i.e., Its direction ratios are proportional to 6, 2, –3.

Now, $\sqrt{6^2+2^2+(-3)^2}=7$

$\therefore$ Its direction cosines are $\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}$.

6. Let E_1 : "A woman is selected", E_2 : "A Hindi knowing person is selected" and E_3 : "A teacher is selected", then $P(E_1)=\frac{20}{50}=\frac{2}{5}$, $P(E_2)=\frac{10}{50}=\frac{1}{5}$, and $P(E_3)=\frac{15}{50}=\frac{3}{10}$

$\therefore$ Required probability $= P(E_1 \cap E_2 \cap E_3)$

$= P(E_1) \times P(E_2) \times P(E_3) = \frac{2}{5}\times\frac{1}{5}\times\frac{3}{10}=\frac{3}{125}$

7. Let $I = \int \frac{x}{(x-1)^2(x+2)}dx$. ...(1)

Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+2}$ ...(2)

$\Rightarrow x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$ ...(3)

Comparing coefficients in (3), we get

$0 = A + C$; $1 = A + B - 2C$; $0 = -2A + 2B + C$

Solving these, we get

$A = \frac{2}{9}, B = \frac{1}{3}, C = -\frac{2}{9}$

$\therefore I = \frac{2}{9}\int\frac{1}{x-1}dx+\frac{1}{3}\int\frac{1}{(x-1)^2}dx-\frac{2}{9}\int\frac{1}{x+2}dx$

$= \frac{2}{9}\log\left|\frac{x-1}{x+2}\right|-\frac{1}{3}\cdot\frac{1}{x-1}+C_1$

OR

$\int \sin x \sin 2x \sin 3x \, dx$

$= \int \sin 3x \sin x \sin 2x \, d.$

$= \frac{1}{2}\int (\cos 2x - \cos 4x)\sin 2x \, dx$

$= \frac{1}{2}\int \sin 2x \cos 2x \, dx - \frac{1}{2}\int \cos 4x \sin 2x \, dx$

$= \frac{1}{4}\int \sin 4x \, dx - \frac{1}{4}\int (\sin 6x - \sin 2x)\, dx$

$= \frac{1}{4}\int \sin 4x \, dx - \frac{1}{4}\int \sin 6x \, dx + \frac{1}{4}\int \sin 2x \, dx$

$= \frac{1}{4}\left[\frac{-\cos 4x}{4} - \frac{(-\cos 6x)}{6} + \frac{(-\cos 2x)}{2}\right] + C$

$= \frac{1}{4}\left[\frac{\cos 6x}{6} - \frac{\cos 4x}{4} - \frac{\cos 2x}{2}\right] + C$

8. We have, $\dfrac{dy}{dx}+2y\tan x=\sin x$

It is linear differential equation of the form

$\dfrac{dy}{dx}+Py=Q$

where $P = 2\tan x$, and $Q = \sin x$

Now, I.F. $= e^{\int 2\tan x\, dx} = e^{2\log|\sec x|} = \sec^2 x$

$\therefore\quad y(\sec^2 x)=\int(\sec^2 x)(\sin x)dx$

$\Rightarrow y(\sec^2 x)=\int \sec x\tan x\,dx$

$\Rightarrow y(\sec^2 x)=\sec x+C$

When $x=\dfrac{\pi}{3}$, $y = 0$

$\therefore\quad (0)[\sec^2(\pi/3)] = \sec(\pi/3) + C \Rightarrow C = -2$

Hence, $y(\sec^2 x) = \sec x - 2$ *i.e.*, $y = \cos x - 2\cos^2 x$ is the required solution.

9. Let $A(2\hat{i}-\hat{j}+\hat{k})$, $B(3\hat{i}+7\hat{j}+\hat{k})$ and $C(5\hat{i}+6\hat{j}+2\hat{k})$

Then, $\overrightarrow{AB}=(3-2)\hat{i}+(7+1)\hat{j}+(1-1)\hat{k}=\hat{i}+8\hat{j}$

$\overrightarrow{AC}=(5-2)\hat{i}+(6+1)\hat{j}+(2-1)\hat{k}=3\hat{i}+7\hat{j}+\hat{k}$

$\overrightarrow{BC}=(5-3)\hat{i}+(6-7)\hat{j}+(2-1)\hat{k}=2\hat{i}-\hat{j}+\hat{k}$

Now, angle (θ) between $\overrightarrow{AC}$ and $\overrightarrow{BC}$ is given by

$\Rightarrow\quad \cos\theta=\dfrac{\overrightarrow{AC}\cdot\overrightarrow{BC}}{|\overrightarrow{AC}||\overrightarrow{BC}|}=\dfrac{6-7+1}{\sqrt{9+49+1}\sqrt{4+1+1}}$

$\Rightarrow\quad \cos\theta = 0 \Rightarrow AC\perp BC$

So, A, B, C are the vertices of right angled triangle.

10. Any point on the given line,

$\dfrac{x-1}{2}=\dfrac{y+1}{-3}=\dfrac{z+10}{8}=k$ (say) ...(i)

is $R(2k+1, -3k-1, 8k-10)$

Since, this is the foot of the $\perp$ from $P(1, 0, 0)$ on (i), then

$(2k+1-1)\cdot 2+(-3k-1-0)\cdot(-3)+(8k-10-0)\cdot 8=0$

$\Rightarrow 4k+9k+3+64k-80=0 \Rightarrow 77k=77\Rightarrow k=1$

$\therefore\quad R$ is $(3, -4, -2)$.

This is the required foot of perpendicular.

Also, perpendicular distance $= PR$

$=\sqrt{(3-1)^2+(-4-0)^2+(-2-0)^2}=\sqrt{24}=2\sqrt{6}$ units.

Also equation of line PR is $\dfrac{x-1}{2}=\dfrac{y}{-4}=\dfrac{z}{-2}$

OR

Given that -6, 3, 4 are intercepts on x, y and z-axes respectively.

$\therefore\quad$ Equation of the plane is $\dfrac{x}{-6}+\dfrac{y}{3}+\dfrac{z}{4}=1$

$\Rightarrow\ -2x+4y+3z-12=0$

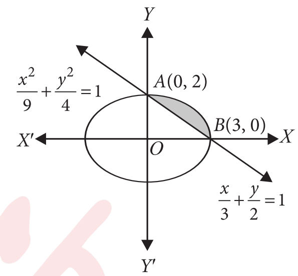
$\therefore\ $ Length of the perpendicular from origin to the plane $-2x+4y+3z-12=0$ is

$\left|\dfrac{-2\times0+4\times0+3\times0-12}{\sqrt{(-2)^2+4^2+3^2}}\right|=\dfrac{12}{\sqrt{29}}$ units

11. We have $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$ ...(i) and $\dfrac{x}{3}+\dfrac{y}{2}=1$ ...(ii)

Curve (i) is an ellipse of the form $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$.

That means its major axis is along x-axis. Also this ellipse is symmetrical about the x-axis.



Required area $=\dfrac{2}{3}\displaystyle\int_0^3\sqrt{(3)^2-x^2}\,dx-\dfrac{2}{3}\int_0^3(3-x)dx$

$=\dfrac{2}{3}\left[\dfrac{x}{2}\sqrt{9-x^2}+\dfrac{9}{2}\sin^{-1}\left(\dfrac{x}{3}\right)\right]_0^3-\dfrac{2}{3}\left[\dfrac{(3-x)^2}{-2}\right]_0^3$

$=\dfrac{2}{3}\left[\left(0+\dfrac{9}{2}\sin^{-1}(1)\right)-\left(0+\dfrac{9}{2}\sin^{-1}(0)\right)\right]+\dfrac{1}{3}[0^2-9]$

$=\dfrac{3\pi}{2}-3$ sq. units.

12. The given line is

$\vec{r}=(2\hat{i}-4\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$

$\Rightarrow\ \dfrac{x-2}{3}=\dfrac{y+4}{4}=\dfrac{z-2}{2}=\lambda$ (say) ...(i)

Any point on it is $(3\lambda+2, 4\lambda-4, 2\lambda+2)$

This lies on the plane $\vec{r}\cdot(\hat{i}-2\hat{j}+\hat{k})=0$

i.e., $x-2y+z=0$ ...(ii)

$\therefore\ 3\lambda+2-2(4\lambda-4)+2\lambda+2=0$

$\Rightarrow\ -3\lambda+12=0 \Rightarrow \lambda=4$

$\therefore\ $ The point of intersection of (i) and (ii) is

$(3\times4+2, 4\times4-4, 2\times4+2)=(14, 12, 10)$

Its distance from the point $(2, 12, 5)$

$=\sqrt{(14-2)^2+(12-12)^2+(10-5)^2}$

$=\sqrt{144+0+25}=\sqrt{169}=13$ units.

13. Consider the following events.

E : Two balls drawn are white

A : There are 2 white balls in the bag

B : There are 3 white balls in the bag

C : There are 4 white balls in the bag

$P(A) = P(B) = P(C) = \frac{1}{3}$

$$P(E/A) = \frac{{}^2C_2}{{}^4C_2} = \frac{1}{6}, \quad P(E/B) = \frac{{}^3C_2}{{}^4C_2} = \frac{3}{6} = \frac{1}{2}$$

$$P(E/C) = \frac{{}^4C_2}{{}^4C_2} = 1$$

$$\therefore \quad P(C/E) = \frac{P(C)\cdot P(E/C)}{P(A)\cdot P(E/A) + P(B)\cdot P(E/B) + P(C)\cdot P(E/C)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1} = \frac{3}{5}$$

OR

Consider the following events:
E_i = Seed chosen is of type A_i, $i = 1, 2, 3$
A = Seed chosen germinates.

We have, $P(E_1) = \frac{4}{10}$, $P(E_2) = \frac{4}{10}$ and $P(E_3) = \frac{2}{10}$

$$P(A/E_1) = \frac{45}{100}, P(A/E_2) = \frac{60}{100}, P(A/E_3) = \frac{35}{100}$$

(a) Required probability = $P(\bar{A}/E_3) = 1 - P(A/E_3)$

$$= 1 - \frac{35}{100} = \frac{65}{100} = 0.65$$

(b) Required probability = $P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)$

$$= \frac{4}{10}\times\frac{45}{100}+\frac{4}{10}\times\frac{60}{100}+\frac{2}{10}\times\frac{35}{100} = \frac{490}{1000} = 0.49$$

(c) Required probability = $P(E_2/\bar{A})$

$$= \frac{P(E_2\cap\bar{A})}{P(\bar{A})} = \frac{P(E_2)P(\bar{A}/E_2)}{P(\bar{A})} = \frac{P(E_2)(1-P(A/E_2))}{1-P(A)}$$

$$= \frac{\frac{4}{10}\times\left(1-\frac{60}{100}\right)}{1-\frac{49}{100}} = \frac{\frac{4}{10}\times\frac{40}{100}}{\frac{51}{100}} = \frac{16}{51}$$

14. (i) Let $I = \int \underset{\text{I}}{x}\ \underset{\text{II}}{\sin 3x}\,dx$

$$= x\int \sin 3x\,dx - \int\left(\frac{d}{dx}(x)\int \sin 3x\,dx\right)dx$$

$$= x\left(-\frac{\cos 3x}{3}\right) - \int 1\left(-\frac{\cos 3x}{3}\right)dx + c$$

$$= -\frac{x\cos 3x}{3} + \frac{1}{3}\int \cos 3x\,dx + c$$

$$\therefore \quad I = -\frac{x\cos 3x}{3} + \frac{1}{3}\cdot\frac{\sin 3x}{3} + c$$

$$= -\frac{x\cos 3x}{3} + \frac{\sin 3x}{9} + c$$

(ii) Let $I = \int \log(x+1)\,dx = \int \underset{\text{I}}{\log(x+1)}\cdot\underset{\text{II}}{1}\,dx$

$$= \log(x+1)\cdot x - \int \frac{1}{x+1}\cdot x\,dx$$

$$= x\log(x+1) - \int\frac{x+1}{x+1}dx + \int\frac{1}{x+1}dx$$

$$= x\log(x+1) - x + \log(x+1) + c$$

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Self Evaluation Sheet

*Once you complete **SQP-4**, check your answers with the given solutions and fill your marks in the marks obtained column according to the marking scheme. Performance Analysis Table given at the bottom will help you to check your readiness.*

Q. No.	Chapter	Marks Per Question	Marks Obtained
1	Integrals	2	
2	Differential Equations / Differential Equations	2	
3	Three Dimensional Geometry	2	
4	Probability	2	
5	Three Dimensional Geometry	2	
6	Probability	2	
7	Integrals / Integrals	3	
8	Differential Equations	3	
9	Vector Algebra	3	
10	Three Dimensional Geometry / Three Dimensional Geometry	3	
11	Application of Integrals	4	
12	Three Dimensional Geometry	4	
13	Probability / Probability	4	
14	Integrals	2×2	
	Total	**40**	....
		Percentage	..............%

Performance Analysis Table

If your marks is

> 90%	TREMENDOUS!	*You are done! Keep on revising to maintain the position.*
81-90%	EXCELLENT!	*You have to take only one more step to reach the top of the ladder. Practise more.*
71-80%	VERY GOOD!	*A little bit of more effort is required to reach the 'Excellent' bench mark.*
61-70%	GOOD!	*Revise thoroughly and strengthen your concepts.*
51-60%	FAIR PERFORMANCE!	*Need to work hard to get through this stage.*
40-50%	AVERAGE!	*Try hard to boost your average score.*